MULTILOOP CONTROL APPLIED TO INTEGRATOR MIMO PROCESSES. A Preliminary Study

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Abstract. This work presents a multiloop control study applied to integrator multiple-input and multiple-output (MIMO) systems. Two PI/PID controller design techniques for MIMO systems are studied -BLT method (Luyben, 1992; Monica et al., 1988) and $\mathcal{H}_\infty$ design according to classical textbook (Skogestad and Postlethewaite, 1996; Zhou and Doyle, 1998)- and, several complications are made evident (specially with the detuning methods (BLT)) when a proportional-integral multiloop control is designed. A discussion together with some recommendations about these problems is included. Also, an industrial application example (Tan *et al.*, 2002) is incorporated in this paper, where the PI controller diagonal matrix is designed in frequency domain following typical recommendations of $\mathcal{H}_\infty$ design. This design is showed in detail so that the reader can see some complications present in these particular MIMO processes.

Keywords: Integrator system, MIMO, Multiloop, PID controllers, $\mathcal{H}_\infty$ design.

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1. Introduction

Multivariable and multiloop control have received much attention in the academic community since several industrial applications have multiple-input and multiple-output variables, such as distillation columns, heat transfer processes and chemical reactors, among others. Thus, multiloop PID controllers are being widely used in the industry because the engineers can control MIMO processes with only a few parameters to tune.

Three types of tuning methods for multiloop control systems are available in the literature; (1) detuning method, (2) sequential closing method and (3) independent design method.

In the detuning method, each controller of the multiloop control system is first designed ignoring process interactions from the other loops. Then, the interactions are taken into account and each controller is detuned until some performance criterion is reached. In this context, the most popular technique of the detuning methods is probably the BLT method (Luyben, 1986 and Monica *et al.*, 1988). However, for the sequential closing method, each controller is designed sequentially. That is, an input-output pair is designed and this loop is closed. Then a second pair is considered while the first controller is closed and so on (Mayne, 1973; Loh et al., 1993). Finally, in the independent design method, each controller is designed based on the paired transfer functions while satisfying some constraints due to the process interactions (Skogestad and Morari, 1989).

In this paper, two tuning methods for multiloop control are studied to control integrator MIMO processes. For this purpose, an attractive and interesting example that shows several problems in the multiloop PI control design is included.

In order to give a systematic exposition of this work, in Section 2 the semi-empirical method known as BLT technique is applied to the integrative MIMO system and a brief discussion related to the example is included. Section 3 comments on a rigorous controller design method (Skogestad and Postlethewaite, 1996) applicable to the integrative MIMO system based on $\mathcal{H}_\infty$ design. Then, an industrial application example is analyzed in Section 4. Finally, in Section 5 the conclusions are presented.

2. Semiempircal Methods

2.1. Biggest log modulus Tuning (BLT)

Detuning techniques for MIMO process control are based on disadjusting the parameters in the controller matrix by using some semi-empirical criteria. BLT method (Luyben, 1986), which is probably the most popular detuning technique (applied to multiloop PI control), uses a detuning factor F for all gains and integrative times of the controllers. Then, Monica *et al.* (1988) extended the BLT method for multiloop PID control following similar ideas to the original method. Both techniques use semi-empirical functions and begin an iterative sequence with PI controllers designed by Ziegler and Nichols' (Z-N) settings (continuous cycling technique). For this reason, the following remark is enunciated:

Remark 1. *If at least one independent control loop in the multiloop system presents infinity gain margin then, BLT methods cannot be applied.*

PROOF

Since BLT methods compute ultimate gain (K_u) and period (P_u), if at least one element of the plant diagonal matrix is stable in an independent control loop with a proportional controller for all $0 < Kc < \infty$ then, it is not possible to compute K_u and P_u and BLT methods cannot be applied.

2.2 An Illustrative Example

It is important to highlight that the BLT methods were not developed for integrator or unstable MIMO processes because the authors never considered these systems (Luyben, 1986 and Monica *et al.*, 1988). One of the purposes of this work is to study this alternative.

Consider a 2x2 system with a transfer function matrix given by

$$G(s)=\begin{bmatrix} \dfrac{e^{-0.5s}}{s(s+1)} & \dfrac{2\,e^{-0.1s}}{3s+1} \\ \dfrac{3\,e^{-0.3s}}{(2s+1)} & \dfrac{4\,e^{-0.2s}}{s(s+3)} \end{bmatrix} \quad . \tag{1}$$

Following the BLT method, two semi-empirical functions are computed. One of them is defined as

$$W(s) := -1 + \det(I + G(s)C(s)) \quad , \tag{2}$$

and the other one is a semi-empirical measurement (L_{cn}) designated by the author as multivariable closed loop log modulus (similar to complementary sensitivity function for SISO systems) defined as

$$L_{cn}(j\omega) := 20\log\left(\left|\frac{W(j\omega)}{1+W(j\omega)}\right|\right) \quad . \tag{3}$$

Luyben (1986) suggests finding the parameters of the PI-controller diagonal matrix by using a detune parameter F such as

$$\|L_{cn}\|_\infty = \max(L_{cn}(j\omega)) = 2n \quad \forall\, 0 \le \omega < \infty \quad , \tag{4}$$

where n is the MIMO system dimension. Thus, the PI controller actions result

$$K_{ci} = K_{ci\text{-}ZN}/F \quad , \tag{5}$$

and

$$T_{Ii} = FT_{Ii\text{-}ZN} \quad , \tag{6}$$

with $i = 1, ..., n$.

For this particular example, it is necessary to find F value such that $\|L_{cn}\|_\infty = 4$ DB due to $G(s)$ is 2x2 system. Figure 1 shows $L_{cn}(j\omega)$ computed in the frequency interval $\Omega = [10^{-2} \;\; 10^{+1}]$ when $F = 8.2345$ is chosen. For this particular case $\|L_{cn}\|_\infty = 4.0000$ DB at $\omega = 0.2000$. Then, it was studied $\|L_{cn}\|_\infty$ versus different F values and this result is shown in Fig. 2. According to this figure there is not only one solution to this particular problem since there are three values for the detuning factor F such that $\|L_{cn}\|_\infty = 4$ DB. Furthermore, there exits an instability region for $0 < F < 2.1$. And, just only for $F = 8.2345$, $\|L_{cn}\|_\infty = 4.000$ DB and the multiloop system is stable. That means that for integrative MIMO systems, the design problem can be non-convex, and a stability condition should be included. While, Luyben and co-workers state that the Eqn. (4) is a non-proofed stability condition, and this example is a counter-example that shows that the condition (4) can fail when the plant transfer function matrix has integrator elements.

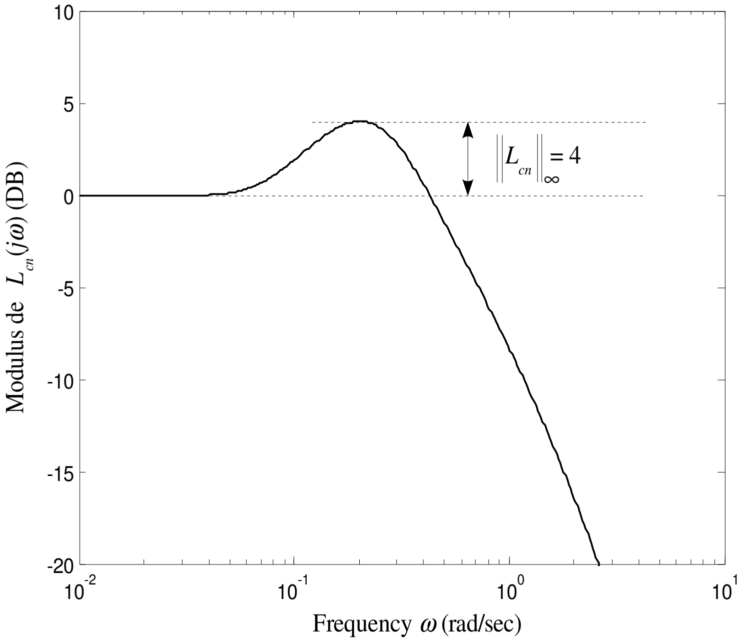

Fig. 1. $L_{cn}(j\omega)$ vs frequency ω with $F = 8.2345$.

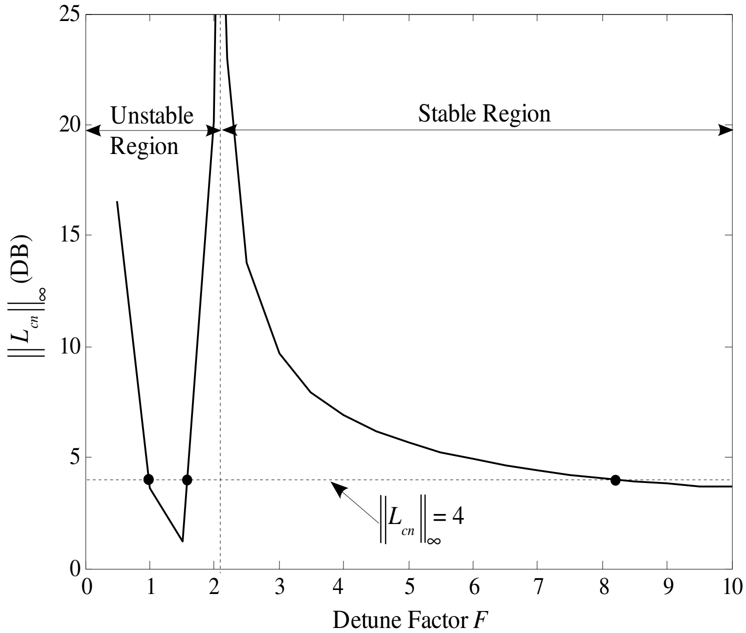

Fig. 2. $\|L_{cn}\|_\infty$ for different F values.

Table 1 summarizes the settings of the controller diagonal matrix and the stability and instability situation for this example.

Table 1. PI controller settings for integrative MIMO system (1).

For three considered cases where $\|L_{cn}\|_{\infty} = 4.0000$.

Detune Factor	Proportional	Integral	Multiloop
$F = 0.9606$	$K_{c1} = 1.0176$	$T_{I1} = 3.8487$	unstable
	$K_{c2} = 1.9439$	$T_{I2} = 1.4205$	
$F = 1.7191$	$K_{c1} = 0.5686$	$T_{I1} = 6.8881$	unstable
	$K_{c2} = 1.0861$	$T_{I2} = 2.5424$	
$F = 8.2345$	$K_{c1} = 0.1187$	$T_{I1} = 32.9916$	stable
	$K_{c2} = 0.3864$	$T_{I2} = 19.3331$	

According to the BLT method (Luyben, 1986), if the designer choices $F = 2$ in other to initializes the detuning procedure, an unstable solution can be found.

3. Mixed Sensitivity Function $\mathcal{H}_{\infty}$ Design

A conceptually simple approach to multivariable control is given by a two-step procedure in which firstly a pre-compensator **W** (for decoupling control) is designed and secondly, a diagonal controller **K** is designed in frequency domain. Invariably this two-step procedure results in a suboptimal design.

With the purpose of quantifying the interaction degree in the MIMO system, the relative gain array (RGA) evaluated in $s = 0$ is traditionally used. Thus, RGA is defined as

$$\mathbf{\Lambda} := \mathbf{G}(0) \mathrm{x} [G(0)^{-1}]^{T} \quad , \tag{7}$$

where x denotes element-by-element multiplication.

Remark 2. *If the transfer function matrix **G**(s) has at least one integrator element then, conclusions about process interaction and input-output pairing by means of traditional RGA are not possible.*

Therefore, when the interaction is strong, the decoupling control must be considered. Thus, decoupling control results when $\mathbf{W}(s)$ is chosen such that $\mathbf{G}_s(s) = \mathbf{G}(s)\mathbf{W}(s)$ is a diagonal matrix. Clearly, this requirement is satisfied when $\mathbf{W}(s) = \mathbf{G}^{-1}(s)$ and the dynamic decoupling is reached. Usually, $\mathbf{G}^{-1}(s)$ results in a non-realizable matrix and in consequence, one alternative for this problem is the steady state decoupling. This alternative is obtained by selecting a constant pre-compensator $\mathbf{W} = \mathbf{G}^{-1}(0)$ resulting $\mathbf{G}_s(0)$ in a diagonal matrix. Note that the following remark for integrator MIMO processes:

Remark 3. *If the matrix **G**(s) has at least one integrator element then, the steady state decoupling is not possible.*

3.1 Design Procedure

The μ-iteration measure by Grosdidier and Morari (1986) is very convenient for a multiloop control system design. However, this measure does not ensure a stable multiloop control system when integrator elements are present in the controllers (integrator modes) or in the plant (one or more integrator transfer functions). And this is the central point of the investigation problem since the integrator elements are present in the plant matrix and in the PI/PID controllers.

The following procedure is based on typical recommendations for the $\mathcal{H}_\infty$ design applicable to multivariable controllers (Skogestad and Postlethewaite, 1996; Zouh and Doyle, 1998):

Step 1. Adopt an objective function as $\mathbf{N} = [\mathbf{W_p S}]$ or $\mathbf{N} = [\mathbf{W_p S} \quad \mathbf{W_u KS}]^T$, where $\mathbf{S}$ is the sensitivity transfer function and, **Wp** and **Wu** are weights function matrix.

Step 2. A reasonable initial choice is to set the weights $\mathbf{Wp} = \text{diag}\{w_{pi}\}$ with $i = 1, 2, ...,$ n, where n is the number of channel and $w_{pi} = (s/M_i+\omega_{Bi})/(s+\omega_{Bi}A_i)$ where $A_i << 1$, $M_i \approx 2$, and ω_{Bi} *is* bandwidth requirement given in mathematical terms as $\bar{\sigma}\left(S(j\omega_B)\right) = 1/\sqrt{2}$, and the performance requirement $|S_i(j\omega)| < 1/|w_{pi}(j\omega)|$ $\forall$ ω. On the other hand, $\mathbf{Wu} = \mathbf{I}$ is usually chosen.

Step 3. Solve the following optimization problem:

$$\min_{\mathbf{K}} \|\mathbf{N}(\mathbf{K})\|_\infty = \min_{\mathrm{K}} \left[\max_{\mathbf{K},\omega} \bar{\sigma}\left(\mathbf{W_p}(\mathrm{j}\omega)\mathbf{S}(\mathrm{j}\omega) \right) \right] \quad , \tag{8}$$

with $\mathbf{K} \in \mathcal{C}$ and $\mathcal{C}$ the stabilizing controller set. In the particular case of this paper, the attention is centered on the PI/PID controller multiloop design. Finally, while the nominal stability is guaranteed in the design of $\mathcal{H}_2/\mathcal{H}_\infty$ optimal controllers by the Youla parameterization procedure (Youla, 1976), here, for a controller with a fixed structure such as PI or PID controllers the nominal stability must be checked.

4. An Industrial Application Example

Consider a boiler with natural recirculation (Tan *et al.,* 2002) where the principal input and output variables are

input:		**output:**	
u_1	feedwater flow rate (Kg/s),	y_1	drum level (m),
u_2	fuel flow rate (Kg/s),	y_2	drum pressure (Mpa),
u_3	attemperator spray flow rate (Kg/s),	y_3	steam temperature (°C).

Tan *et al.* (2002) identifies the following LTI model using experimental input-output data and the MATLAB *System Identification Toolbox*:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) & g_{13}(s) \\ g_{21}(s) & g_{22}(s) & g_{23}(s) \\ g_{31}(s) & g_{32}(s) & g_{33}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{(-0.16s^2+0.052s+0.0014)10^{-3}}{(s^2+0.0168s)} & \frac{(3.1s-0.032)\,10^{-3}}{(s^2+0.0215s)} & 0 \\ \frac{-0.0395\,10^{-3}}{(s+0.018)} & \frac{2.51\,10^{-3}}{(s+0.0157)} & \frac{(0.588s^2+0.2015s+0.0009)10^{-3}}{(s^2+0.0352s+0.000142)} \\ \frac{(-0.00118s+0.000139)}{(s^2+0.01852s+0.000091)} & \frac{(0.448s+0.0011)}{(s^2+0.0127s+0.000095)} & \frac{(0.582s-0.0243)}{(s^2+0.1076s+0.00104)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{9}$$

The normal setpoints for this operation are

$$\boldsymbol{y_{sp}} = \begin{bmatrix} y_{1\mathrm{sp}} \\ y_{2\mathrm{sp}} \\ y_{3\mathrm{sp}} \end{bmatrix} = \begin{bmatrix} 1.00 \\ 6.45 \\ 466.7 \end{bmatrix} , \tag{10}$$

and, the model constraints of the manipulated and controlled variables are not taken into account in this paper.

Note three important observations in this example.

1. The element $g_{22}(s)$ in the transfer function matrix (9) is a first order. For this reason, it is impossible to calculate a K_u and P_u with a proportional controller because it presents infinity gain margin. In consequence, BLT methods cannot be applied (remark 1).
2. Clearly, the system is coupled, but the traditional RGA evaluated in s = 0 is not possible to apply due to the integrator elements in the plant matrix (9) (remark 2).
3. According to the previous topic, the steady state decoupling is not applicable in this example (remark 3).

For the reasons stated above, three PI control feedback loops (Fig. 3) were implemented without pre-compensator for decoupling the system, and the control technique enunciated in Section 3.1 was chosen for the design.

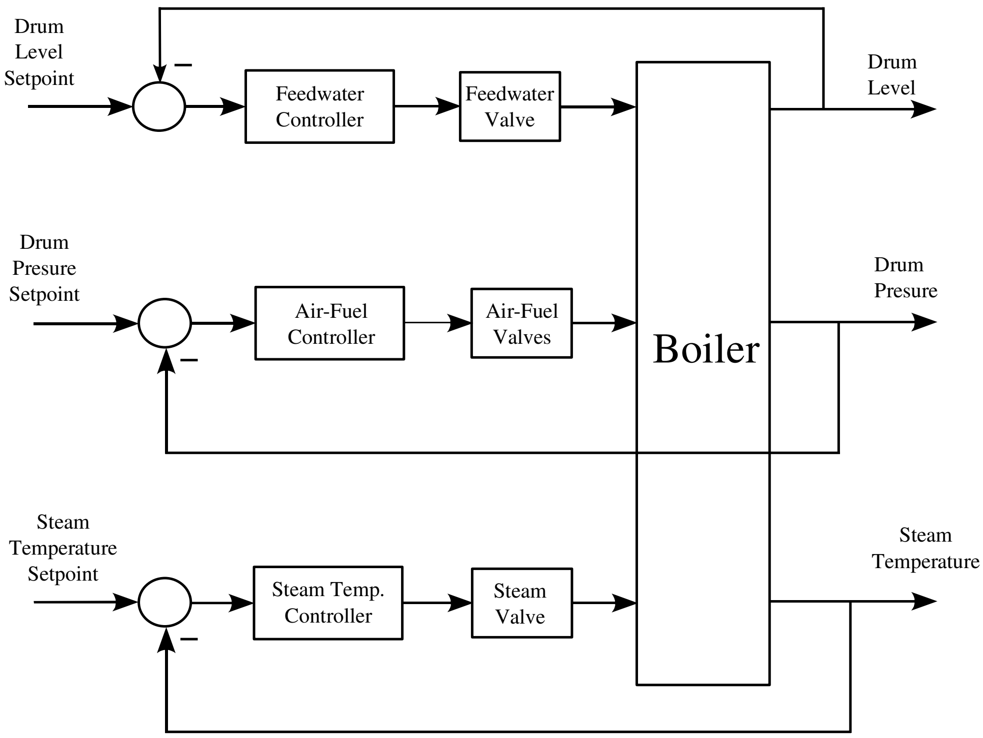

Fig. 3. Classical PI feedback control loop for the utility boiler.

Following the procedure detailed in Section 3.1,

Step 1. It was adopted $\mathbf{N} = [\mathbf{W_p S}]$, where $\mathbf{S} = [\mathbf{I} + \mathbf{GK}]^{-1}$.

Step 2. By simplicity in a preliminary design, it was chosen $\mathbf{W_p} = \text{diag}\ \{w_{pi}\}$ where $w_{pi} \geq 1/S_{ii}$ with $S_{ii} = 1/(1 + g_{ii}k_i)$ and k_i a PI controller given by $k_i(s) = K_{ci}(1 + 1/T_{Ii}s)$ tuned via IMC-PID controller settings (Morari and Zafiriou, 1989). This adoption of w_{pi} is based on the classical recommendation present in the robust control literature (Skogestad and Postlethewaite, 1996; Zhou and Doyle, 1998; among others).

Step 3. The optimization problem (8) was solved with $\mathbf{K} = \text{diag}\{k_i\}$.

According to the IMC technique, each controller was tuned as,

$$q_i = \frac{1}{g_{ii}^{0\,MP}} f_i \quad , \tag{11}$$

with i = 1, 2, and 3, and where f_i represents an IMC filter (first-order filters in this example), g_{ii}^{0MP} is the invertible part (minimum phase portion) of g_{ii}^{0}, and g_{ii}^{0} is an element of the diagonal matrix (9), where the superscript 0 denotes the nominal transfer function. Then, the PI controllers were computed as

$$k_i = \frac{q_i}{1 - g_{ii}^{0} q_i} \quad . \tag{12}$$

Thus, following Morari and Zafiriou recommendations, the adopted PI controller actions result

$$K_{ci} = \tau_{ii}/K_{pii}\lambda_i \quad , \tag{13}$$

and

$$T_{Ii} = \tau_{ii} \quad , \tag{14}$$

with i = 1, 2 and 3. The constants K_{pii} and τ_{ii} are summarized in Table 2, and λ_i is the IMC filter time constant. Note that, it is easy to reach to Eqns. (13) and (14) with the IMC procedure for the nominal plant $g_{22}(s)$. But, some algebraic considerations must be taken into account for $g_{11}(s)$ and $g_{33}(s)$.

Table 2. Parameters involved in the Eqn. (13) and (14) in order to calculate the PI controller actions.

Controller	K_{pii}	τ_{ii}
1	$-8.333\ 10^5$	59.52
2	0.16	63.69
3	23.37	93.14

According to the IMC approach, λ_i is a tuning parameter in each canal. Thus, the problem basically consists of finding the set of parameters $\Lambda = \{\lambda_1, \lambda_2, \lambda_3\}$ such that $\|\mathbf{N}\|_\infty$ is minimized. Using computational techniques in the frequency domain for this particular example with the set of parameters $\Lambda = \{10000, 500, 30\}$, the $\|\mathbf{N}\|_\infty$ = 18.4691 is reached. Figure 4 shows $|\mathbf{N}|$ in the frequency domain and the $\|\mathbf{N}\|_\infty$ = 18.4691[1] at ω = 0.0100. It is important to notice that the plant matrix (9) is ill-conditioned due to the nature of the variables involved; in consequence the input-output scaling is not good. For this reason, the computational technique convergence is not easy to reach.

1 A similar value was reported by Tan et al. (2002).

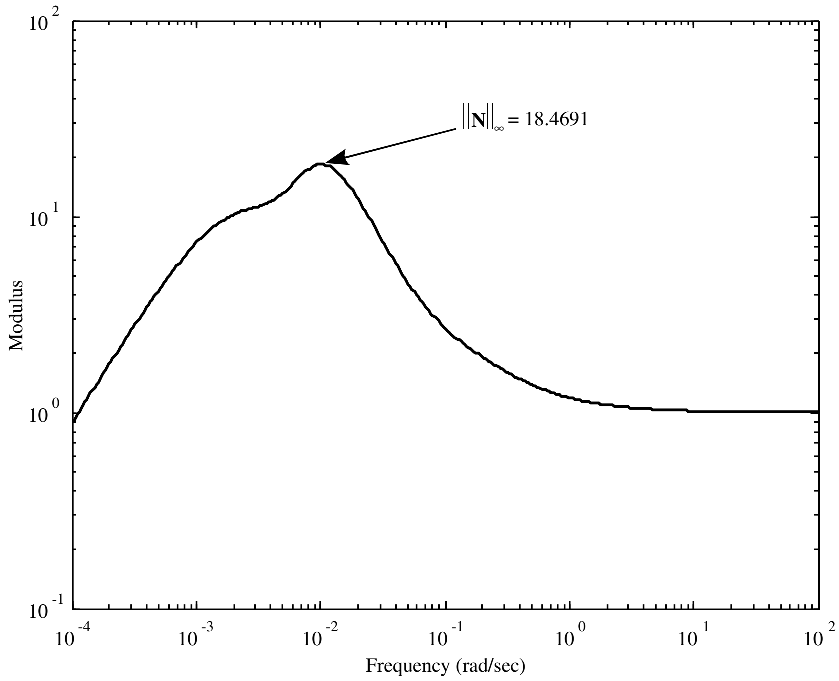

Fig. 4. Modulus of **N** vs. frequency.

Table 3 summarizes the PI controller settings implemented in the multiloop control system suggested in Fig. 3.

Table 3. PI controller settings for multiloop control of Fig. 3.

Controller	K_{ci}	T_{Ii}
1	71.43	59.52
2	0.8	63.69
3	-0.13	93.14

Figures 5, 6 and 7 show the step responses of the boiler with setpoint changes in drum level, drum pressure and steam temperature using PI controller diagonal matrix. According to Figs. 5 and 6 it is possible to observe that the drum level and the pressure are linked and this behavior is due to a mixture of liquid and gas in the boiler drum.

Figure 5 shows that a level step change has serious impact on the controlled temperature and a little effect on the drum pressure. While, according to Fig. 6 a step change in the pressure has an effect on the liquid-gas level inside the boiler drum and a strong impact on the steam temperature.

Figure 7 shows the effect on the drum level and the pressure when a steam temperature step change is introduced. In this case, a small interaction is observed.

Also, according to Figs. 5 and 7 the drum level and the steam temperature setpoint changes have little effect on the drum pressure.

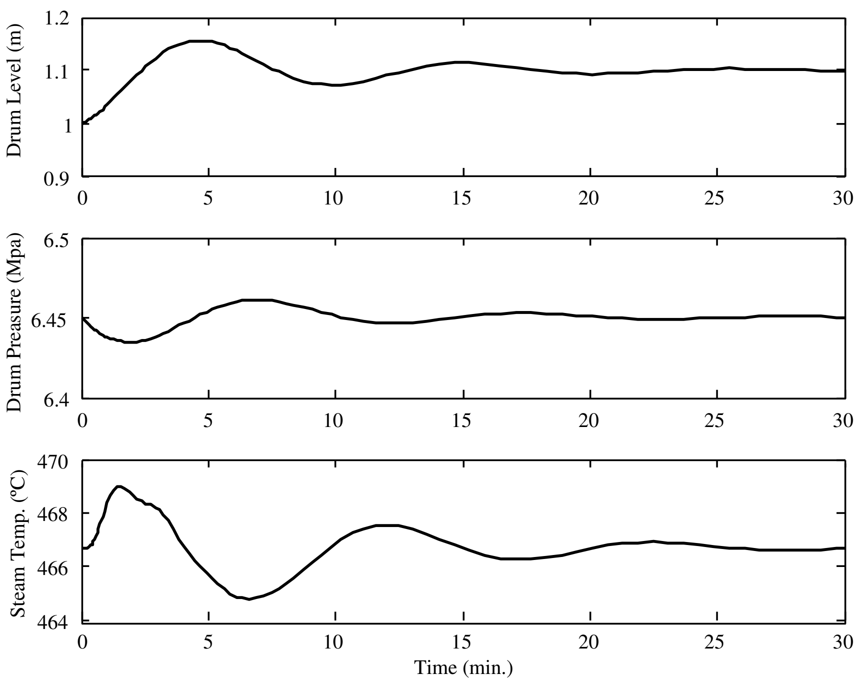

Fig. 5. Boiler time responses for a 10% drum level increase with the designed PI controllers designed according to the procedure indicated above.

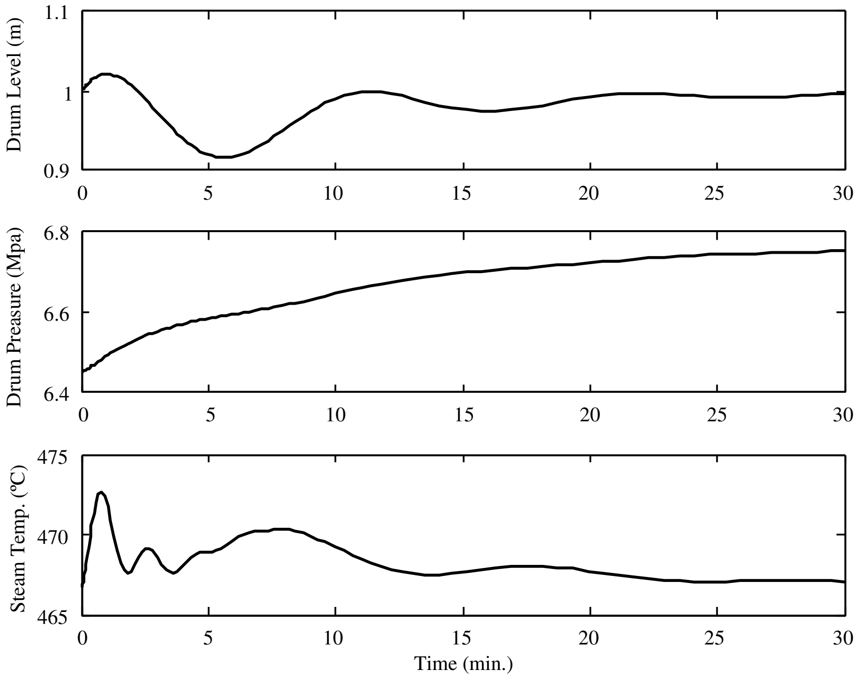

Fig. 6. Boiler time responses for a 5% drum pressure increase with the designed PI controllers designed according to the procedure indicated above.

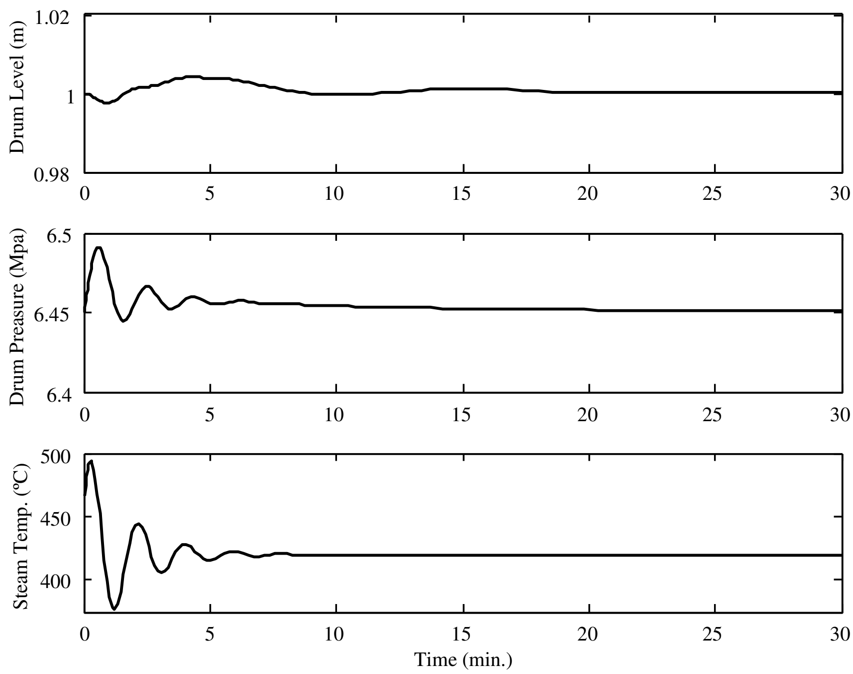

Fig. 7. Boiler time responses for a 10% steam temperature decrease with the designed PI controllers designed according to the procedure indicated above.

Clearly, let's remark that according to the simulation results (Figs. 5, 6 and 7) that, i) the input-output pairing, feedwater – drum level is linked with drum pressure and steam temperature when a drum level change is introduced while, ii) air-fuel – drum pressure is strongly linked with drum level and steam temperature when drum pressure change is introduced, and iii) small effects are observed when a steam temperature change is introduced, indicating that there is a slight interaction of variables in this direction.

5. Conclusions

Several complications with integrator MIMO processes were detected when the multiloop control with PI/PID controller diagonal matrix is designed. From the analysis of the results, it is possible to conclude that for integrator MIMO processes

1) the traditional BLT detuning method can fail since an unstable solution can be found,
2) the traditional RGA and steady state decoupling can not be applied and
3) $\mathcal{H}_\infty$ controller design technique shows to be the adequate method for these cases.

However, the $\mathcal{H}_\infty$ design applied to PI/PID controllers is a challenging problem specially if i) the plant transfer function matrix is ill-conditioned and includes integrator elements, and ii) the designer wants to include a pre-compensator.

Thus, a most comprehensive study with RGA in frequency domain would let us understand the interaction level more clearly and a pre-compensator must be taken into account. Clearly, if a pre-compensator designed at a particular frequency is introduced in the multiloop PI/PID controller design, a better performance in time domain can be reached.

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