AADECA 2006 – XXº Congreso Argentino de Control Automático
28 al 30 de Agosto de 2006 - Buenos Aires, Argentina.

OFFSET ELIMINATION USING RECEDING HORIZONT CONTROLLERS: A COMPREHENSIVE ANALYSIS

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Abstract: An offset free control is one that drives the controlled outputs to their desired targets at steady state. In the linear model predictive control (MPC) framework, the elimination of steady state offset may seem a little obscure, since the closed loop optimization tends to hide the integral action. Theoretically, implementing a well posed optimization problem and having unbiased prediction in steady state are sufficient conditions to eliminate offset output. However, these basic conditions are not always achieved in practical applications. This paper presents a detailed practical analysis of the existing strategies to eliminate offset when linear models with moderated uncertainties are controlled. In addition, these strategies are used for a Continuous Stirred Tank Reactor (CSTR) control with nonlinear dynamic, in order to evaluate the formulations in a more realistic process.

Keywords: MPC, Predictive Control, CSTR Temperature Control, Linear Control.

1. INTRODUCTION

Model predictive control (MPC) refers to a class of control algorithms that optimizes future plant behavior through the use of an explicit mathematical model. At each sample time, the controller input is taken as the first element of an open-loop optimal input sequence that is computed by driving the model predicted outputs as close as possible to a desired future trajectory. At each sample time, the system state is estimated and a new open-loop optimization is carried out.

MPC technology is widely-used in chemical process industries where it is generally the default technology for advanced process control applications. In practice, modeling error and unmeasured disturbances can lead to steady-state offset unless precautions are taken in the control design. Elimination of steady-state offset is accomplished in two basic ways. The first approach involves working with models in their velocity form, that is, models which use input changes and state changes instead of input and state values. These models permit a well posed optimization problem since the targets of the state increments are always correct (i.e., they are always zero) even if the plant and the model are not equal (Pannochia, 2001). An uncommon example of the implementation of this kind of models to MPC, can be seen in Rodrigues and Odloak (2003), and Odloak (2005). In these works, the integral action is achieved by using the inputs in the incremental form in both, the output predictions and the observer.

The second method involves an augmented process model including a constant step disturbance. This disturbance, which is estimated from the measured process variables, is generally assumed to remain constant in the future and its effect on the controlled variables is removed by shifting the steady-state target for the controller. The most widely-used industrial MPC implementations, such as DMC, QDMC, and IDCOM-M use a constant output step disturbance model to achieve offset-free control. However, while this method has proved to be acceptable for stable plants, it does not work for unstable systems because the observer poles contain the unstable poles of the process model. To overcome this problem, more general state-space models were developed in the literature (Muske and Badgwell, 2002; Pannochia and Rawlings, 2003), that allow considering input, state and output disturbances, and can handle unstable plants.

In this paper, we focus attention on the analysis of the main existing strategies to eliminate output offset, and elucidate the critical point of some algorithms that, contrary to the appearances, cannot lead to an offset free MPC. In addition, a comprehensive comparison between the performance of the different approaches is performed through a few numerical simulations.

The organization of this work is as follows. Section 2 includes a theoretical framework presentation related to three popular MPC formulations. Then, section 3 presents numeric simulations that show the improvement reached in the performance with those three techniques. Finally, in Section 4, the conclusions are summarized.

2. THEORETICAL FRAMEWORK

As mentioned before, two conditions are sufficient to obtain an offset-free control. The first one consists of getting an unbiased prediction of the steady state, which

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is achieved by using integral action in the observer. This detail, in spite of its simplicity, is not always evident in the MPC formulation since the integral action is often (incorrectly) attributed to the use of velocity form models in the optimization problem. Note that, if the observer does not reach accurately the stationary states – i.e., if the observer does not include an integral action – then, the predictions are made based on a mistaken stationary model. Therefore, the optimization will lead the wrong system, not the accurate one, to the set points. As can be seen, this behavior produces a steady state error since, in general, the inputs that eliminate the wrong-model offset are different from the ones that eliminate the offset in the true plant.

The second condition obliges to design a well posed MPC optimization problem, and can be succinctly explained as follows (J. A. Rossiter, 2003). Suppose that the observer gets unbiased stationary estimations, but the optimization problem is not well posed (the steady state minimum does not correspond to zero tracking error). Since the performance index is not set up so that the minimum (at steady state) corresponds to zero tracking error, then the converse must occur; that is, the optimum control will unavoidably cause offset. This is what happens when absolute input values (not the increments) are used in the cost function and the desired output is different from zero.

In the following paragraphs, we describe standard strategies and expose the way each one accomplishes the above conditions.

2.1. Maciejowski's Velocity Form

Consider the original state space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \qquad (1)$$

where “x”, “u” and “y” represent the state, the input and the model output respectively, and A, B and C are matrices of appropriate dimension.

A typical MPC formulation (that we call Strategy 1) is based on the following cost function:

$$\begin{aligned} V_1\big(x(k),k\big) \triangleq \sum_{j=0}^{p} \big[Cx(k+j/k) - y_{sp} \big]^T \\ Q\big[Cx(k+j/k) - y_{sp} \big] + \sum_{j=0}^{m-1} \Delta u(k+j)^T R \Delta u(k+j) \end{aligned} \qquad (2)$$

where “p” and “m” are the prediction and control horizon respectively, “y_{sp}“ stands for the output set points, $\Delta u(k) = u(k) - u(k-1)$ are the input increments, which at the same time are the optimization variables, and Q and R are positive definite weighting matrices. This cost function is minimized subject to

$u_{\min} \leq u(k+j) \leq u_{\max}$,

$\Delta u_{\min} \leq \Delta u(k+j) \leq \Delta u_{\max}$,

and the successive states (predictions) are computed using the current measured state $x(k)$ and the following velocity model[2]:

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix} &= \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u(k) \\ y(k) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} \end{aligned} , \qquad (3)$$

The objective function (2) together with velocity models like (3), produce a well posed optimization problem, since the combination $y = y_{sp}$ and $\Delta u = 0$ is always possible at steady state.

Figure 1 shows an MPC close-loop diagram in which the block called “o*ptimizer*” is the one that performs the minimization described above (Maciejowski 2002). In this diagram, matrices Υ and Ψ accommodate the prediction equations taking into account model in (3). Note that in (3), the input “u” represents itself a new state, which would not need to be estimated by the observer but computed by saving the implemented value. This seems to be right, since estimating a variable which was implemented one sampling time before, makes no much sense. However, as it is shown next, this is the reason why this control structure produces output offset.

The trouble arises because the integral action must be performed explicitly by the observer. Assume that for a time $\bar{k}$ large enough, the system reaches the steady state. At this time, the output predictions will be

$$\begin{aligned} \hat{y}\big(\bar{k}+j/\bar{k}\big) &= CA^j \hat{x}_{\bar{k}} \\ &+ C\big(I + A + \cdots + A^{j-1}\big) Bu_{\bar{k}} \qquad 1 \leq j \leq p \end{aligned} , \qquad (4)$$

where $\hat{y}\big(\bar{k}+j/\bar{k}\big)$ represents the predicted output, $\hat{x}_{\bar{k}}$ represents the estimated state (that remains constant for stable observers), and $u_{\bar{k}}$ is the steady state input. Note that, since we assume steady state conditions, then $\Delta u_{\bar{k}} = 0$. On the other hand, the observer steady state equation is given by[3]

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + L_{aux}\big[y_{\bar{k}} - C\big(A\hat{x}_{\bar{k}} + Bu_{\bar{k}}\big)\big] , \qquad (5)$$

where L_{aux} is the observer gain, and $y_{\bar{k}}$ is the (measured) output feedback. From this equation, we can see that there is not reason for the model output, $C\big(A\hat{x}_{\bar{k}} + Bu_{\bar{k}}\big)$, to achieve the plant output, $y_{\bar{k}}$. Consequently,

$$\hat{x}_{\bar{k}} \neq A\hat{x}_{\bar{k}} + Bu_{\bar{k}} . \qquad (6)$$

So, the steady state prediction equation will be given by

$$\begin{aligned} \hat{y}\big(\bar{k}+1/\bar{k}\big) &= C\big(A\hat{x}_{\bar{k}} + Bu_{\bar{k}}\big) \neq y_{\bar{k}} \\ &\vdots \\ \hat{y}\big(\bar{k}+p/\bar{k}\big) &= CA^p \hat{x}_{\bar{k}} + C\big(I + A + \cdots + A^{p-1}\big) Bu_{\bar{k}} \neq y_{\bar{k}}, , \end{aligned} \qquad (7)$$

which means that it will not be possible to drive the outputs to their set points. This is a case in which, despite the optimization problem is well posed, the output predictions are not accurate enough.

The natural way to overcome this trouble is by adding an integrating mode to the observer. This can be made in two different forms: by including the complete model (3), that is, estimating the input “u” together with the

[2]Note that this augmented model includes an integrating modes.

[3]Equation (4) is derived from a typical discrete state observer.

original states; or by adding a disturbance model. Using the former of these alternatives, which we call Strategy 2, it is easy to see that the steady state observer equation is given by

$$\begin{bmatrix} \hat{x}_{\bar{k}} \\ \hat{u}_{\bar{k}} \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{\bar{k}} \\ \hat{u}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u_{\bar{k}} + \begin{bmatrix} L_x \\ L_u \end{bmatrix} \left[y_{\bar{k}} - \begin{bmatrix} C & 0 \end{bmatrix} \left(\begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{\bar{k}} \\ \hat{u}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u_{\bar{k}} \right) \right] \quad \textbf{(8)}$$

where L_u and L_x form the observer gain matrix[4], and again, $\Delta u_{\bar{k}} = 0$. This equation leads to

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}} + L_x \left(y_{\bar{k}} - \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{\bar{k}} \\ \hat{u}_{\bar{k}} \end{bmatrix} \right) \quad \textbf{(9)}$$

$$\hat{u}_{\bar{k}} = \hat{u}_{\bar{k}} + L_u \left(y_{\bar{k}} - \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{\bar{k}} \\ \hat{u}_{\bar{k}} \end{bmatrix} \right). \quad \textbf{(10)}$$

From (10) we see that, if L_u is of full rank, then

$$y_{\bar{k}} = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}} \\ \hat{u}_{\bar{k}} \end{pmatrix} = C\left(A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}} \right); \quad \textbf{(11)}$$

and from (9),

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}}. \quad \textbf{(12)}$$

Finally, from (11) and (12), equations (7) will produce accurate outputs predictions.

It is then clear that the use of "$\hat{u}(k)$" instead of "$u(k)$" will eliminate the steady state model mismatch. In addition, we observe that, in general, is $\hat{u}_{\bar{k}} \neq u_{\bar{k}}$, which means that the additional state is only a fictitious variable with no physical meaning.

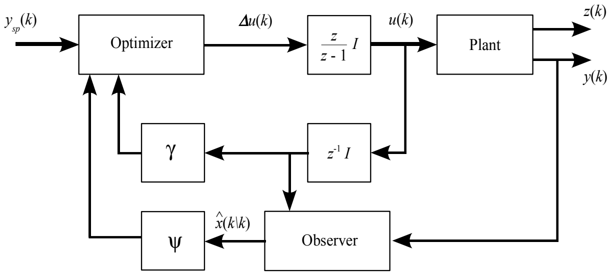

Figure 1: A representative block diagram of the Maciejowski's control structure.

2.2. Complete Velocity Form

A different strategy to have an offset free controller is by using the complete velocity form (Prett et al., 1988).
This kind of models considers the increments on both, the input and the states, and has the following form:

$$\begin{aligned} \varsigma(k+1) &= \tilde{A}\varsigma(k) + \tilde{B}\Delta u(k) \\ z(k) &= \tilde{C}\varsigma(k) \end{aligned} \quad \textbf{(13)}$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}, \ \tilde{B} = \begin{bmatrix} B \\ CB \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} 0 & I \end{bmatrix}, \ \varsigma(k) = \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix}$$

$$\Delta x(k) = x(k) - x(k-1).$$

Based on this model, the MPC cost function can be written as

$$V_2(k) \triangleq \sum_{j=0}^{p} \left(\varsigma(k+j/k) - \bar{y}_{sp} \right)^T Q \left(\varsigma(k+j/k) - \bar{y}_{sp} \right) + \sum_{j=0}^{m-1} \Delta u(k+j)^T R \Delta u(k+j)$$

where $\bar{y}_{sp} = \begin{bmatrix} 0 & \cdots & 0 & y_{sp} \end{bmatrix}^T$. Observe at this point, that the steady state output predictions will be

$$\hat{y}(\bar{k}+j/\bar{k}) = \tilde{C}\tilde{A}^j \hat{\varsigma}(\bar{k}) = \hat{y}(\bar{k}), \qquad 1 \le j \le p$$

which means that the predicted output will remain constant and equal to the corresponding observer estimation. Now, taking into account the integrating mode in (13) it is easy to see that, at steady state, the estimated output will reach the measured output, and the offset-free condition of this control structure becomes evident. In order to organize the numeric simulation section, we call this approach, Strategy 3.

2.3. Linear Regulator with disturbance sub-model

A general linear regulator structure (Rawlings 2000) is shown in Fig. 2. The MPC regulator block uses the following cost function:

$$V(k) \triangleq \sum_{j=0}^{\infty} \bar{x}(k+j/k)^T Q \bar{x}(k+j/k) + \sum_{j=0}^{m-1} \bar{u}(k+j)^T R \bar{u}(k+j) \quad \textbf{(14)}$$

where

$$\begin{aligned} \bar{x}(k+1) &= A\bar{x}(k) + B\bar{u}(k) \\ \bar{x}(k) &= \hat{x}(k) - x_s \quad , \\ \bar{u}(k) &= u(k) - u_s \end{aligned}$$

and x_s and u_s represent the state and input target respectively. The ability of this strategy to eliminate output offset depends exclusively on the target calculation stage. In order to obtain unbiased target values, the observer performs the estimations based on the following general disturbance model

$$\begin{bmatrix} x(k+1) \\ d(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} A & G_d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \\ p(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} C & 0 & G_p \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \\ p(k) \end{bmatrix},$$

where $d(k)$ and $p(k)$ represent the state and the output disturbances respectively, and G_d and G_p are the model matrices that determine the effects of the disturbances on the states and the output. In this way, the steady state observer equation can be written as

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}} + L_x\left[y_{\bar{k}} - C\left(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}} \right) - G_p\hat{p}_{\bar{k}} \right] \quad \textbf{(15)}$$

[4]Observe that the observer defined in (8), in strategy 2, is different from the one defined in (5). The input "u", is now estimated.

$$\begin{bmatrix} \hat{d}_{\bar{k}} \\ \hat{p}_{\bar{k}} \end{bmatrix} = \begin{bmatrix} \hat{d}_{\bar{k}} \\ \hat{p}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} L_d \\ L_p \end{bmatrix} \left[y_{\bar{k}} - C\left(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d \hat{d}_{\bar{k}} \right) - G_p \hat{p}_{\bar{k}} \right] \quad \textbf{(16)}$$

Assuming now that the system is detectable and $\left[L_d^T \;\; L_p^T \right]^T$ is of full rank, we have

$$y_{\bar{k}} = C\left(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d \hat{d}_{\bar{k}} \right) + G_p \hat{p}_{\bar{k}} .$$

Then, from (15),

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d \hat{d}_{\bar{k}} ,$$

which finally implies

$$y_{\bar{k}} = C\hat{x}_{\bar{k}} + G_p \hat{p}_{\bar{k}} .$$

This means that, if the disturbances $d(k)$ and $p(k)$ are taken into account in the target calculation and the system reaches a stationary value, then, the predicted output will match the plant output. Finally, as the optimization problem that minimizes (14) is well posed – i.e., the cost can be zeroed - then, the steady state output offset will be eliminated. This strategy (named strategy 4 in this work) can handle both, input and output set-points. In the case that the output set-points can not be achieved by means of the input set-points, the state and input targets are obtained from an optimization problem. This problem takes the form:

$$\min_{x_s, u_s} \left\{ \left(y_{sp} - y_t^a \right)^T Q_s \left(y_{sp} - y_t^a \right) + \left(u_s - u_{sp} \right)^T R_s \left(u_s - u_{sp} \right) \right\}$$

subject to:

$$x_s = Ax_s + Bu_s + G_d \hat{d}\left(k \right)$$

$$y_t^a \triangleq Cx_s + G_p \hat{p}\left(k \right)$$

where y_t^a represents the achievable stationary outputs, and x_s, u_s are the targets passed to the regulator stage. If y_{sp} is achievable, then $y_t^a = y_{sp}$. Note that this strategy must solve two optimization problems at each sampling time "k".

Finally, some general comments must be made about the linear regulator algorithm.

- Note that the regulator stage in Fig. 2 only drives the system to the state and input targets, and does not include disturbance sub-models. This is a way to separate the dynamic and the stationary parts.
- Despite the augmented model presented above is a general way to describe disturbances entering the process, a list of conditions must be accomplished in order to make the model detectable (otherwise, it would be no possible to construct an stable observer). Muske and Badgwell 2002 describe these conditions when model (A,B,C) has integrating and stable modes; in our case, however, it is only necesary to know that the total number of augmented disturbance states must not exceed the number of outputs.
- The augmented disturbance states are not controllable by the inputs u (Pannocchia and Rawlings 2003). However, if the augmented system is detectable, they are used to remove their influence form the controlled variables.

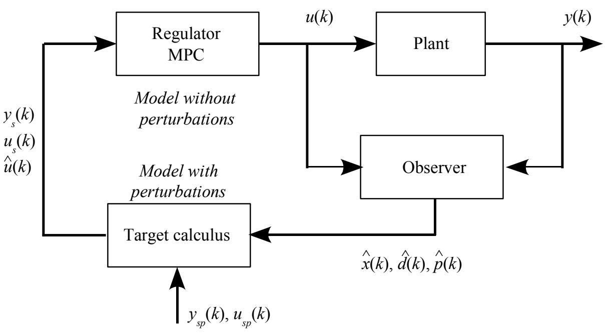

Fig. 2: A representative block diagram of the predictive control system with linear regulator (Strategy 4).

3. NUMERIC SIMULATIONS

In this section, two problems are simulated that show the behavior of the previous formulations. Note that this verification must be done in the presence of model uncertainties, otherwise the offset problem does not appear. First, a linear case with gain uncertainty is simulated to observe the offset elimination when a step change in the set point and the load variables are introduced. Then, a non-linear case - a CSTR with significant parametric uncertainty - is tested, where the results confirm the expected offset elimination property.

3.1. Linear Case

Consider a linear plant given by,

$$G(s) = \frac{K(-9s+1)}{45s^2+18s+1} , \quad \textbf{(17)}$$

where $K = 1$ for the real plant, and $K^0 = 0.85$ for the nominal plant. In addition, an output load variable is included in order to consider set point and load changes.

Figure 4 shows the step response (time interval [50, 200]) when the Strategy 1 and the corresponding reformulation (Strategy 2) are implemented. Note that in the Maciejowski's form the offset is not null after the transitory response has ended, while the second strategy has completely eliminated the error. The responses to the load change show a behavior consistent with the previous one.

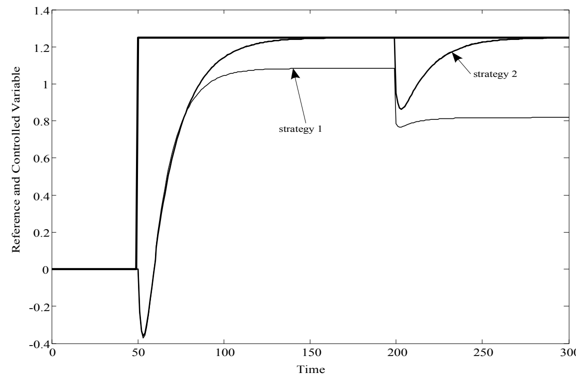

Fig. 4: Step responses at set point and load changes when the Strategy 1 and 2 are implemented.

On the other hand, Fig. 5 shows the time response given by strategies 2, 3 and 4 to changes in set point and load. In the three cases, the offset is completely eliminated.

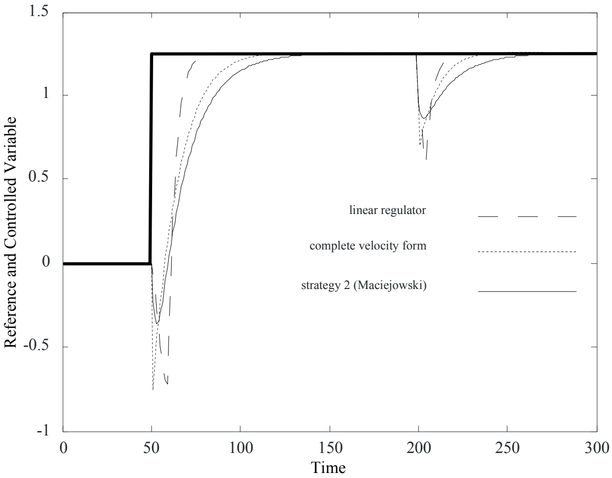

Fig. 5: Step responses to set point and load changes when the strategies 2, 3 and 4 are implemented.

The inspection of the results in Fig. 5 tells us that all the discussed strategies produce quite reasonable offset free responses. In particular, the linear regulator seems to be more appropriate than the other ones when the settling time is taken as a critical feature of the desired response. Note that a fair comparison between the analyzed treatments should involve an study of the individual tunning procedures, which is out of the scope of this paper.

3.2. Non-Linear Case

Consider a CSTR with a strong nonlinear dynamic described in the Appendix 1. The goal is to control the reactor temperature in presence of set-point changes and unexpected disturbances (feed temperature, feed concentration, process flow rate, cooling flow rate and inlet cooling temperature). Figure 6 shows the output responses of the strategies 1 and 2 when a step set point change is introduced. Clearly, strategy 1 gives offset while the strategy 2 reaches the new reference signal.

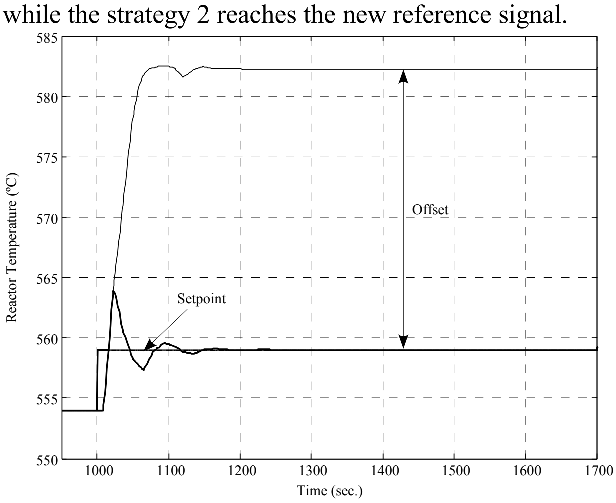

Fig. 6: Step responses to set point changes when the strategies 1 and 2 are implemented.

Figure 7 shows time responses of the strategies 1 and 2 when load step changes (10% in Q_i^0 and 10% in T_i^0) are introduced. Again, the strategy 1 gives offset whereas the strategy 2 completely rejects the load changes.

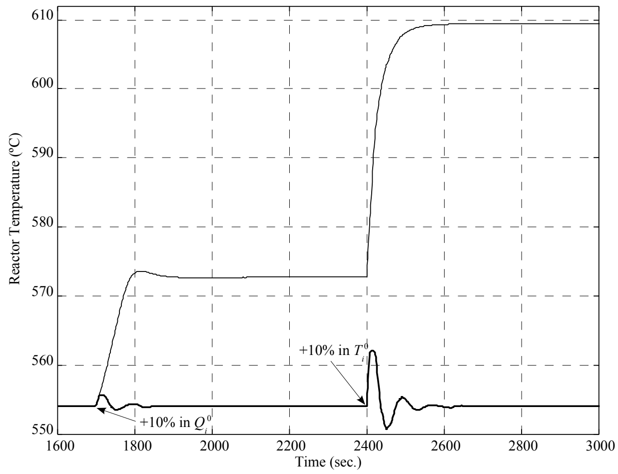

Fig. 7: Time responses using strategies 1 and 2 when load step changes (10% in Q_i^0 and 10% in T_i^0) are introduced.

Figure 8 shows the time responses reached for strategies 2, 3 and 4. Clearly, the three strategies eliminate the output offset when a set point change is introduced. The same result is verified when a load change (10% in Q_i^0) is introduced into the CSTR (Figs. 9).

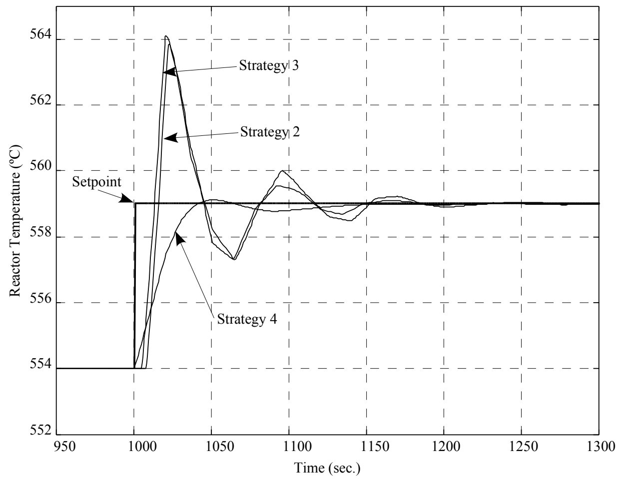

Fig. 8: Step response of the strategies 2, 3 and 4.

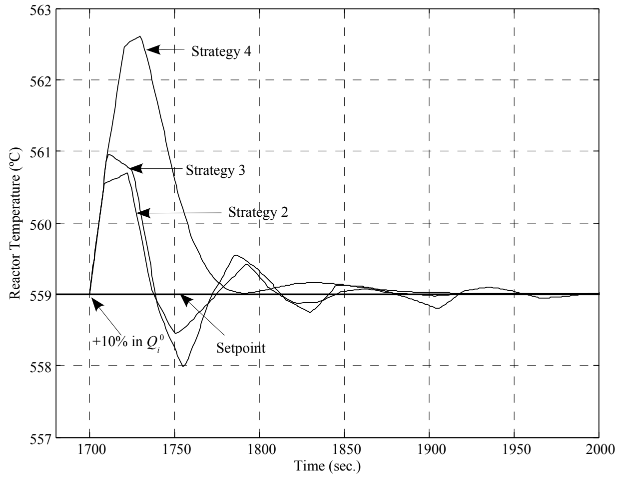

Fig. 9: Time response of the strategies 2, 3 and 4 when a load step change (10% in Q_i^0) is introduced.

In this example it is not posible to use both kind of disturbances $d(k)$ and $p(k)$ simultaneusly when strategy 4 is implemented, since there is only one output. Following the detectability conditions, the choice $G_p = 1$ and $G_d = [\ ]$ shows to be appropiate to achieve the main objective of eliminating output offset. Note that, since the MPC regulator does not use the augmented model (it is only used by the observer and the target calculation), the output performance is not strongly dependent of matrices G_p and G_d.

4. CONCLUSIONS

Based on the results obtained from running different simulation examples, it is possible to conclude the following:

(i) an adequate reformulation of the Maciejowski's velocity-form model can lead to a free offset time response.

(ii) offset free MPC controllers are not always simple to design, particularly when velocity-form models (strategy 2) are used to predict the output behavior. In this case, the augmented state must be estimated, despite it represents a variable that can be measured.

(iii) the resulting controller yields offset-free responses only in case that the prediction is unbiased (which implies the use of an observer with integrating mode) and the optimization problem is well posed. These are important features to be considered when designing a MPC controller.

5. APPENDIX

A1. Non Linear Reactor Model

Consider the CSTR where an exothermic and irreversible chemical reaction is carried out. The reactive **A** generates the product **B** while the reactor content is being cooled by an appropriate coolant fluid. Here, it is assumed that the reactor dynamic involves the following four states: volume, reactive concentration, reactor temperature and cooling temperature inside the jacket.

The reactor dynamic is modeled by the following equations:

$$\frac{dV}{dt} = q_i - q_o \quad , \tag{18}$$

$$\frac{d(Vc_A)}{dt} = q_i\, c_{Ai} - q_o\, c_A - Vr \quad , \tag{19}$$

$$\rho c_p \frac{d(VT)}{dt} = q_i\, \rho\, c_p\, T_i - q_o\, \rho\, c_p\, T_o + (-\Delta H) V r_A - UA(T - T_{co}) \quad . \tag{20}$$

$$\rho c_{pc} \frac{d(V_c T_c)}{dt} = q_{ci}\, \rho_c\, c_{pc}\, T_{ci} - q_{co}\, \rho_c\, c_{pc}\, T_{co} + UA(T - T_{co}) \tag{21}$$

Furthermore, it must be noted that

i. the reactor volume must be evaluated as $V = A_T h$, where A_T is the transversal section of the tank and it is considered constant,
ii. the heat transfer section A results, $A = 2\pi r_T h$ where r_T is the tank radius.
iii. The nonlinear model is completed with the kinetic reaction rate $r_A = kc_A$ where $k = k_0 e^{-E/RT}$,
iv. the product flow rate is defined as $\hat{q}_s = \hat{h}/R_h$, where the super index ^ indicates this is a deviation variable referenced to an steady-state value and R_h is the hydraulic resistance of the valve.

The nominal values of model parameters and main variables are presented in Table 1. They are based on data given by Aris and Amundson (1958).

Table 1. Nominal CSTR Parameter Values.

Parameter	Nomenclature	Value
feed concentration	C_{Ae}	0.5 mol m^{-3}
reactor concentration	C_A	mol m^{-3}
process flow rate	q_e	0.0200 m^3 s^{-1}
feed temperature	T_e	690 K
inlet coolant temperature	T_c	298 K
coolant flow rate	q_{ci}	0.014 m^3 s^{-1}
CSTR volume	V	1 m^3
CSTR level	h	1.27 m
heat-transfer term	U	10.57 Kcal m^{-2}s^{-1}K^{-1}
reaction rate constant	k	-1.08 10^{+16}s^{-1}
activation energy	E/R	2.2645 10^{+4}K^{-1}
heat of reaction	ΔH	-9885 Kcal Kmol^{-1}
liquid density	ρ	60 Kg m^{-3}
specific heat	c_p	1 Kcal Kg^{-1}K^{-1}

A simple test was applied to determine the parameters of a linear transfer function between the controlled temperature and coolant flow rate. This test consisted of introducing a step change in the flow rate q_{ci} (the manipulated variable) equal to 10% of initial value and then the reactor temperature response was registered vs. time. The parameters of the transfer function relating the reactor temperature and the coolant flow rate were computed using a multi parametric optimization algorithm from a toolbox of Matlab. This procedure gave the following result: a second order transfer function with parameters K = -2253.52, T_1 = 3.97 and T_2 = 12.59.

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