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Adaptive Iterative Learning Control Applied to Nonlinear Batch Reactor

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Abstract— **This work presents an application example of adaptive iterative learning control (ILC) applied to nonlinear batch reactor with constrains in the manipulated variable. The strong nonlinearities of the plant can lead to a non-monotonic convergence of the l_2-norm of the error, and still worse, a unstable equilibrium signal $e_\infty(t)$. By numeric simulation this works shows that with the adaptive ILC is possible to obtain a better performance in the controlled variable than with the traditional feedback and, with the feedback based-ILC.**

Keywords— **ILC, Batch Reactor, Linear Control, Adaptive Control.**

I. INTRODUCCIÓN

ILC associates three interesting concepts. *Iterative* refers to a process that executes the same setpoint trajectory over and over again. *Learning* refers to the idea that by repeating the same thing, the system should be able to improve the performance. Finally, *control* emphasizes that the result of the learning is used to control the plant. For this reason, ILC constitutes the adequate theoretical framework to study new control alternatives for the batch process due to the repetitive nature of the operation.

Several authors obtain interesting results when the ILC scheme is implemented in real processes (Arimoto *et al.*, 1984; Lee and Lee, 1997; Lee *et al.*, 1999, among others). However, when the nonlinearities are strong, ILC can reach unsatisfactory results. For example, Adam (2006) shows that when feedback based ILC is implemented to control a batch reactor, the performance with the controlled variable can be poor, obtaining a non-monotonic convergence of the l_2-norm of the error. Similar results were showed by Amann (1996), Amann *et al.* (1996) and, Owens and Hätönen (2005) who proposed to include an optimal learning algorithm to achieve a reduction of the l_2-norm of the error at each trail.

On the other hand, the idea of combining adaptive control with ILC was presented by several authors (Chien and Yao, 2004; Tayebi, 2004; among others) especially with robotics applications, outside of chemical engineering research. This paper present an adaptive ILC scheme applied to a batch reactor with acceptable results where the l_2-norm of the error is reduced at each trail and an almost monotonic convergence is achieved.

The organization of this work is as follows. Section II included a theoretical framework presentation related to adaptive ILC scheme here studied. Then, Section III presents by means of numeric simulations the behavior of the batch reactor in closed loop when the designer pretends to apply the adaptive ILC linear theory to a nonlinear system. Finally, in Section IV the conclusions are summarized.

II. THEORETICAL FRAMEWORK

A. The Basic Idea of the Adaptive ILC.

The ILC scheme was initially developed as a feedforward action applied directly to the open-loop system (Arimoto, 1984; Kurek and Zaremba, 1993; among others). However, if the system is integrator or unstable to open loop, or well, it has wrong initial condition, the ILC scheme to open loop can be inappropriate. Thus, the feedback-based ILC has been suggested in the literature as a more adequate structure (De Roover, 1996; Moon *et al.* 1998; Doh et al., 1999; Tayebi and Zaremba, 2003).

In this work, a traditional self-tuning regulator (STR) is combined with feedback-based ILC and, the basic idea is shown in Fig. 1.

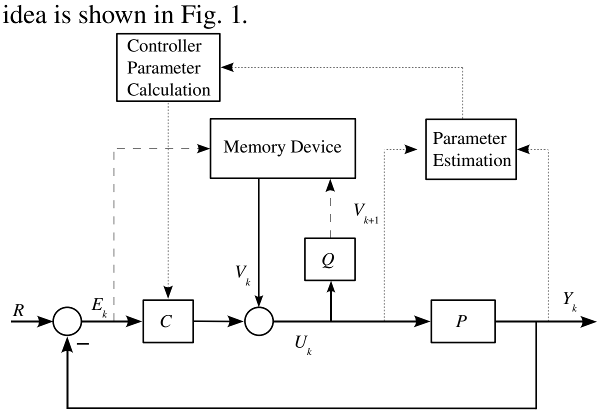

Figure 1. Schematic diagram of STR combined with feedback-based ILC. Here, continuous lines denote the signals used during the *k*-th trail, dashed lines denote

signals will be used in the next iteration and doted lines belong to STR scheme.

This scheme operates as follows. Consider a plant, which is operated iteratively with the same setpoint trajectory over and over again, as a robot or an industrial batch process. During the *k-th* trail an input-signal $u_k(t)$ is applied to the plant, producing the output signal $y_k(t)$. Both signals are stored in the memory devise. Thus, two vectors with length T_f are available for the next iteration. If the system of Fig. 1 operates to open loop, using $u_k(t)$ in the k+1-*th* trail it is possible to obtain the same output again. But, if the k+1 iteration includes $u_k(t)$ and $e_k(t)$ information then, new $u_{k+1}(t)$ and $y_{k+1}(t)$ can be obtained. The importance of the input-signal modification is to reduce the tracking error as the iterations are progressively increased. That is, $\| e_{k+1} \| \leqslant \| e_k \| \ \forall\ k \geqslant 0$. Thus, the purpose of an ILC algorithm is to find a unique equilibrium input signal $u_\infty(t)$ which minimizes the tracking error.

Due to the existing strong nonlinearities in the chemical systems, the ILC scheme by itself cannot lead to a monotonic decrease of the error (in a great number of cases). For such reason, an adaptive scheme is added in order to obtain a stable decreasing error at each trail as it shows in the next section. The STR scheme here implemented follows the traditional recommendations given by classical authors as Isermann (1981), Astrom and Wittenmark (1989), among others.

B. The Tracking ILC Formulation

The ILC formulation uses an iterative updating formula and the most common algorithm suggested by several authors (Arimoto, 1984; Horowitz, 1993; Bien and Xu; 1998; Tayebi and Zaremba, 2003; among others) for the control signal is given by,

$$V_{k+1} = Q(V_k, + CE_k), \tag{1}$$

where $V_1 = 0$, C denotes the controller transfer function and Q is a linear filter[1].

Six postulates were originally formulated by different authors (Chen and Wen, 1999; Norrlöf, 2000; Scholten, 2000, among others) those are:

1. Every iteration ends in a fixed time of duration T_f.
2. Plant dynamics are invariant throughout the iterations.
3. The set-point, $r(t)$ with $t \in [0, T_f]$ is given a priori.
4. For each trail the initial states are the same. That means that $x_k(0) = x_0(0)$, $k \in \mathbb{Z}^+$.
5. The plant output $y(t)$ is observable.
6. There exists a unique input, $u_\infty(t)$, that yields the desired output, $r(t)$, with a minimum tracking error $e_\infty(t)$.

A major issue in ILC is the convergence, and each type of ILC has its own convergence criterion.

1 In this paper variables in time domain are denoted with small letters and variables in s-domain are denoted with capital letters.

The tracking error $e_k(t)$ is defined as

$$e_k(t) := r(t) - y_k(t) \quad , \tag{2}$$

where the subscript k denotes the run number and e_k represents (finite-length) output error trajectory for *k-th* trail.

The idea is to find a input trajectory u_k that minimizes the output error,

$$\| e_k \| \rightarrow \quad \gamma = \min_u \|e\| \quad . \tag{3}$$

as $k \rightarrow \infty$, where $\| \ \ \|$ is some vector norm.

Clearly, γ is a inferior level to be reached by feedback based-ILC as k-index is increased.

Definition 1. *The feedback based-ILC system is said to have monotonic convergence if*

$$\forall\ k \geqslant 0: \quad \gamma \leq \| e_{k+1} \| \leq \| e_k \| \ . \tag{4}$$

Then, the tracking error $e_\infty(t)$ is an equilibrium signal reached by the control system if the system has this error signal for all future trails.

Definition 2. *The equilibrium signal $e_\infty(t)$ is said to be stable if*

$$\forall B > 0, \exists b > 0, \left\| e_0(t) - e_\infty(t) \right\| < b \quad => \\ \forall k \geqslant 0, \left\| e_k(t) - e_\infty(t) \right\| < B \ , \tag{5}$$

where $e_0(t)$ is the initial tracking error.

Definition 3. *An equilibrium signal $e_\infty(t)$ is said to be asymptotically stable if it is stable and*

$$\exists b > 0, \left\| e_0(t) - e_\infty(t) \right\| < b \quad => \\ \lim_{k \rightarrow \infty} \left\| e_k(t) - e_\infty(t) \right\| = 0 \ . \tag{6}$$

The definitions presented before can be founded in the literature (Bien and Xu, 1998; Norrlöf 2000).

1) A Simple Iterative Updating Formula.

Now, consider the iterative updating formula (1) and according to Fig. 1, $U_k = V_k + CE_k$. Then,

$$V_{k+1} = QU_k \quad . \tag{7}$$

Adam (2005) shows that,

$$E_{k+1} = S(1 - Q)R + SQE_k \tag{8}$$

and,

$$U_{k+1} = \frac{T}{P} R + SQU_k \quad . \tag{9}$$

with S and T the sensitivity and complementary sensitivity functions.

Also, based on Tayebi and Zaremba (2003), Adam (2005) proofs the following remark for LTI plant without model uncertainty:

Remark 1. *Consider a feedback-based ILC scheme in Fig. 1 with the updating formula* (7) *and the plant is a LTI system without model uncertainty. If there exists*

C(s) such that the nominal stability is satisfied, then by adopting Q such that $\| SQ \|_\infty \leq 1$ *the tracking error is reduced as k is increased and it is bounded for all* $k \in \mathbb{Z}^+$ *and converges uniformly to*

$$e_\infty(t) = \lim_{k\to\infty} e_k(t) = \mathscr{L}^{-1}\left(\frac{S(1-Q)}{1-SQ}R\right) \quad , \tag{10}$$

when $k \to \infty$ *in the sense of the* l_2*-norm.*

According to Eq. (8), for $k \to \infty$,

$$E_\infty = S(1-Q)R + SQE_\infty \tag{11}$$

or well,

$$E_\infty = \frac{S(1-Q)}{1-SQ}R \quad . \tag{12}$$

Similarly and according to Eqs. (9),

$$U_\infty = \frac{T}{P}R + SQU_\infty \tag{13}$$

or well,

$$U_\infty = \frac{T}{P(1-SQ)}R \quad . \tag{14}$$

Based on Eqs. (12) and (14) the following remark can be enunciated (Adam, 2005):

Remark 2. *Consider the feedback-based ILC scheme in Fig. 1 with the updating formula* (4) *and the plant is a LTI system without model uncertainty. If there exists C(s) such that the nominal stability is satisfied, then by adopting* $Q = 1$ *the perfect control can be reached as* $k \to \infty$.

Based on the last remarks the following design procedure is enunciated:

Design Procedure 1 (Nominal Case):

Step 1: Adopt a controller such that nominal stability and performance are satisfied.

Step 2: Set $Q = 1$ or well $Q(s)$ to be low pass filter such that, $Q(\omega) \to 1\ \forall\ \omega \in [0,\omega_c]$, and $Q(\omega) \to 0\ \forall\ \omega > \omega_c$ with ω_c a cut-off frequency.

Step 3: Use the ILC updating formula (1) or (4).

C. The Implemented Adaptive Scheme

In this work it is proposed to combine an on-line parameter identification of the plant with feedback-based ILC strategy. The classical literature presents two schemes clearly different to implement adaptive control, one of these is i) the Model reference adaptive control (MRAC) and the other one is ii) the self-tuning regulator (STR). In this work, due to the necessity to obtain on-line process data for the implementation of the ILC, it took advantage of these data to implement a STR scheme. Thus, for this reason, a direct adaptive control was implemented.

Notice that, in the block diagram of Fig. 1 it is possible to distinguish three blocks related to: i) data acquisition and parameters estimation of the plant, ii) adaptation mechanism for the controller design and iii) the controller with autotuning parameters.

As for the identification, the algorithms used for the on-line parameter estimation are the extreme importance. Here, it is considered that the system is perfectly deterministic and there are no disturbances and noises.

Now, consider the model,

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_1 u(k-d-1)$$
$$+ b_2(k-d-2) + \cdots + b_n u(k-d-n) \quad , \tag{15}$$

it is possible to write in a vectorial form,

$$y(k) = \varphi^{\mathrm{T}}(k)\,\theta \quad , \tag{16}$$

where,

$$\varphi^{\mathrm{T}}(k) = [-y(k-1), -y(k-2), \cdots, y(k-n), u(k-d-1), \cdots,$$
$$u(k-d-n)] \quad , \tag{17}$$

and,

$$\theta = [a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n] \quad . \tag{18}$$

Then, $2n$ parameters must be found, and in consequence, $2n$ data of $u(k)$ and $y(k)$ are necessary. Thus, a linear equation system can be written where a_i and b_j are unknown parameters. That is,

$$y(k) = \varphi^{\mathrm{T}}(k)\,\theta \quad ,$$
$$y(k+1) = \varphi^{\mathrm{T}}(k+1)\,\theta \quad ,$$
$$\vdots \qquad \vdots$$
$$y(k+N-1) = \varphi^{\mathrm{T}}(k+N-1)\,\theta \quad , \tag{19}$$

or well,

$$Y_k = \Psi_k\,\theta \quad , \tag{20}$$

where $N = 2n$, $\Psi_k = [\varphi^{\mathrm{T}}(k), \varphi^{\mathrm{T}}(k+1), \cdots, \varphi^{\mathrm{T}}(k+N-1)]^T$ and $Y_k = [y(k), y(k+1), \cdots, y(k+N-1)]^T$.

Then, the solution of Eq. (20) is given by,

$$\theta = \Psi_k^{-1}\, Y_k \quad . \tag{21}$$

Finally, based on the estimated parameters, it is possible to tune the controller parameters following a criterion for controller design. In this work, IMC criterion was implemented following the Morari and Zafiriou (1989) recommendations.

III. ADAPTIVE FEEDBACK BASED-ILC APPLIED TO BATCH REACTOR

In this section, the non-linear batch reactor control with strong parametric uncertainty is studied by means of numeric simulation using adaptive feedback based-ILC presented in previous section.

A. Non-Linear Batch Reactor

Consider a batch reactor with a nonlinear dynamic where an exothermic and irreversible second order chemical reaction **A** $\rightarrow$ **B** takes place. It is assumed that the reactor has a cooling jacket whose temperature can be directly manipulated. The goal is to control the reactor temperature by means of inlet coolant temperature. Furthermore, the manipulated variable has minimum and maximum constrains. That is, $T_{cmin} \leq T_c \leq T_{cmax}$, T_{cmin} = -10, T_{cmax} = 20 and, T_c is written in deviation variable.

In order to clarify the understanding of this work, the dynamic equations and the nominal values of the batch reactor are included in this section. For a more detail and explanation it is suggested to see Adam (2006).

The reactor dynamic is modeled by the following equations:

$$\frac{dc_A}{dt} = -k_0 e^{-ER/T} c_A^2 \quad , \tag{22}$$

$$\frac{dT}{dt} = -\frac{\Delta H}{Mcp} k_0 e^{-ER/T} c_A^2 - \frac{UA}{Mcp}(T - T_c) \quad . \tag{23}$$

Also, it must be noted that the reaction rate kinetic is $r_A = kc_A^2$ with $k = k_0 e^{-E/RT}$ and the nominal batch reactor values are summarized in Table 1.

Table 1. Nominal batch reactor values.

parameter	nomenclature	value
feed concentration	c_{Ae}	0.9 mol m^{-3}
feed temperature	T_e	298.16 K
inlet coolant temperature	T_c	298.16 K
	UA/Mcp	0.0288 l min^{-1}
reaction rate constant	k_0	4.7 10^{+19} l mol^{-1}s^{-1}
activation energy	E/R	13550 K^{-1}
	$\Delta H/Mcp$	-5.79 K l mol^{-1}

Adam (2006) computed the transfer function parameters using a Matlab optimization toolbox based on a multiparametric optimization algorithm and nominal values resulted to be K = 1, T = 1.4370. Also, the author shows that the transfer function structure of the batch reactor changes according to operation point, demonstrating the nonlinearities of the system.

B. The Adaptive PI Implemented

For the particular problem described in previous section, Adam (2006) implemented a PI controller with fixed parameters tuned by Morari and Zafiriou (1989) recommendations. Thus, the controller parameters result to be,

$$Kr = T/(K\,\lambda) \quad , \tag{24}$$

$$T_I = T \quad , \tag{25}$$

where K_r and T_I denote the proportional gain and the integral action time respectively and, λ is the IMC filter time constant.

Now, considering the transfer function of the reactor indicated in the previous section then, the Eq. (15) has two parameters to estimate, that is, a_1 and b_1. In consequence, the vectors $\varphi^T(k)$, Ψ_k ,and θ result to be,

$$\varphi^T(k) = [-y(k-1),\ -u(k-1)] \quad , \tag{26}$$

$$\Psi_k = [\varphi^T(k),\ \varphi^T(k+1)] \quad , \tag{27}$$

and

$$\theta = [a_1, b_1]^T \quad . \tag{28}$$

Finally, based on a_1 and b_1 it is possible to calculate K and T by means of the following expressions,

$$K = b_1/(1 - a_1) \quad , \tag{29}$$

and

$$T = -T_s/\ln a_1 \quad . \tag{30}$$

where T_s is the sample time.

Thus, the following procedure was implemented:

<u>Design Procedure 2:</u>

Step 1. Using sample data, compute $\varphi^T(k)$ and Ψ_k according to Eqs. (26) and (27).

Step 2. Compute θ with (21) and, a_1 and b_1 with Eq. (28).

Step 3. Compute K and T by means of (29) and (30).

Step 4. Finally, compute the controller parameters K_r and T_I according to Eqs. (24) and (25).

C. Numeric Simulations

The batch reactor has operation sequence divided in three stages, start-up, run and shutdown. The controlled temperature inside the reactor is monitored during these three stages and, the adaptive feedback based-ILC scheme was implemented by means of the combination of the design procedures 1 and 2.

Figure 2 shows the controlled temperature performance obtained by a traditional feedback and it is compared with the one obtained by means of adaptive feedback-based ILC implementation. For the numeric simulations, according to the previous section, a PI controller was tuned by IMC parameterization (Morari and Zafiriou, 1989). It is possible to distinguish that the controlled temperature can follow the reference with a good exactitude when the adaptive feedback based-ILC is implemented. Furthermore, the reader can note that there is not strong difference as k is increased.

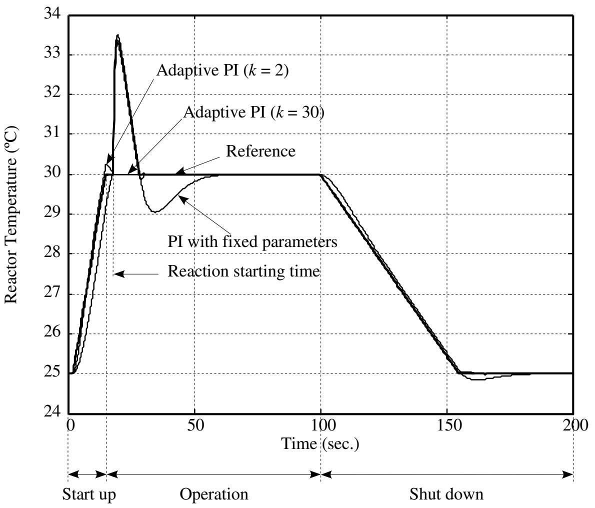

Figure 2. Controlled temperature inside the batch reactor when traditional feedback and adaptive feedback based-ILC are implemented.

Figure 3 shows the dynamic errors obtained with the three cases presented in the Fig. 2. Clearly, the dynamic error is considerably smaller when the adaptive feedback based-ILC is implemented.

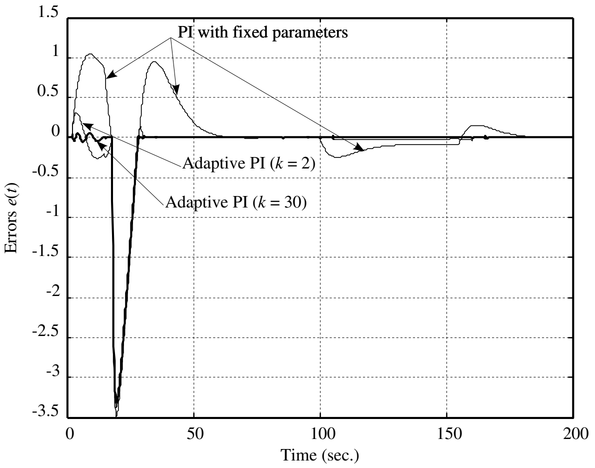

Figure 3. Dynamic errors for the cases studied in Fig. 2.

From a practical point of view, the error is practically zero in almost all the time interval, excepting a small interval associated to the reaction starting time. Neither the traditional feedback control nor the adaptive feedback based-ILC can reject that disturbance due to the saturation of the manipulated variable. This phenomenon is showed in Fig. 4. Notice that when the reaction begins both control schemes try to correct the increase of temperature in the reactor, but quickly the manipulated variable is saturated and as consequence, the peak of temperature observed cannot be avoided. On the other hand, outside of the time interval of the manipulated variable saturation, the correction of adaptive feedback based-ILC is better than the traditional feedback by a better use of the manipulated variable.

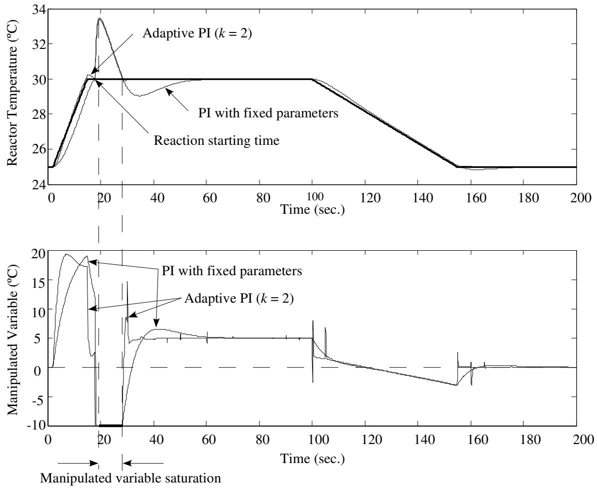

Figure 4. Temperature reaction response and manipulated variable for the traditional feedback and adaptive feedback based-ILC during the second iteration.

Figure 5 compares the l_2-norm[2] ratio between dynamic errors obtained with the ILC schemes and the traditional feedback as a function of the iteration index k. Two ILC schemes were used, 1) a feedback based-ILC implemented with PI controller with fixed parameter (Adam, 2006) and, 2) a feedback based-ILC implemented with an adaptive PI controller according to the Section III.B. Here $\| e \|_{2,k}$ denotes the l_2-norm of the error obtained with the k-iteration while, $\| e \|_{2,0}$ denotes the l_2 -norm of the error obtained with the traditional feedback with PI controller with fixed parameter.

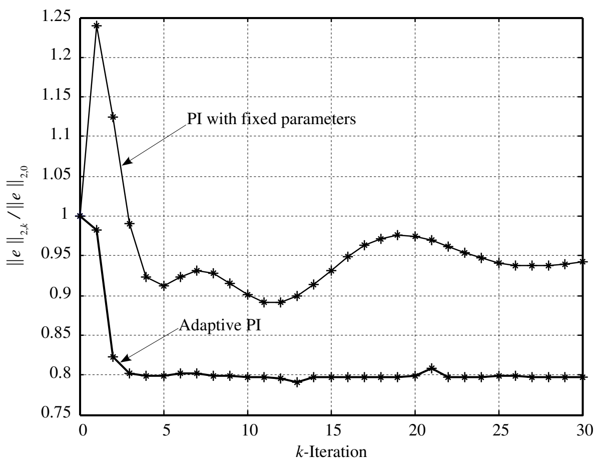

Figure 5. Ratio between $\| e \|_{2,k}$ and $\| e \|_{2,0}$.

Notice that, the feedback based-ILC scheme with PI controller with fixed parameter does not have a

2 The l_2-norm refers to the Euclidean norm defined in the traditional form.

monotonic convergence of the $\| e \|_2$. On the contrary, when the adaptive feedback based-ILC is implemented an almost monotonic convergence of the $\| e \|_2$ is reached. Only in few points, the requirement $\| e_{k+1} \|_2 \leqslant \| e_k \|_2 \; \forall \; k \geqslant 0$ is not fulfilled but a stable decreasing error is reached. Nevertheless, the maximum possible performance and the equilibrium signal are next to be achieved with few iterations.

Finally, notice that, the tracking equilibrium error does not indeed tend to zero as $k \rightarrow \infty$ and, it is associated to manipulated variable saturation during a small time interval.

IV. CONCLUSIONS

Based on the results from different numeric simulations, it is possible to conclude that with the adaptive feedback-based ILC implemented in this reactor, the control system can reach the maximum possible performance (in practical terms) with few iterations. On the contrary, if the feedback-based ILC is implemented alone, a stable equilibrium signal with a monotonic terminal convergence will be little probable, especially if the non-linearities of the system are considerably strong.

The methodology of combining the STR scheme with feedback-based ILC has showed to be an attractive alternative for chemical engineering problems, at first, with good results.

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