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Designing and tuning robust feedforward controllers

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Abstract

A model-based design and tuning procedure is proposed for feedforward controllers, which accounts for model uncertainties in SISO systems. Proper relationships for the analysis of feedforward–feedback control systems show that tuning the feedforward controller is not completely independent from the feedback-loop spectral characteristics. Similarly, fine tune of the associated feedback controller requires to be based on the residual disturbance remaining after the feedforward control action. Consequently, the simultaneous tuning is proposed for efficiently solving disturbance-rejection problems. Two application examples show that the robust combined tuning gives satisfactory results for different dynamics and different tuning requirements.

Keywords: Feedforward controller; Feedforward–feedback systems; Model-based control; $\mathcal{H}_\infty$ analysis

1. Introduction

Classical methods for tuning feedback controllers like the earliest Ziegler and Nichols (1942) and Cohen and Coon (1953) have been used for decades. Ziegler–Nichols uses experimental measurements of the ultimate gain and the ultimate period to calculate the controller settings based on a quarter-decay criterion. Cohen–Coon uses the same tuning criterion assuming that a first-order-plus-delay process model is available.

More recently, Rivera and co-workers (1986) introduced a PID controller design method based on internal model control (IMC) that is attractive to industrial users because it has only one tuning parameter. Later, Chien and Fruehauf (1990) extended this approach to cover three different implementations of the PID algorithm and a wide range of process models. Despite of these and other contributions to the feedback-tuning problem, the level of performance obtained can be insufficient when a disturbance strongly affects the control system.

There are many examples from different engineering areas where a disturbance input has a strong effect on the controlled variable. Mainly, they are cases where the control variable is slower than the disturbance variable because of a large time constant or a large time delay. When this disturbance is measurable, the use of a combined feedforward–feedback control system is advisable, and in fact, very common in the process industry: it can be found in distillation columns (Rix, Löwe, & Gelbe, 1997), power plants (Weng & Ray, 1997), and continuous reactors among many other examples. The extensive application of this combined scheme moved some researchers to initiate investigations in this area. Sternad and Söderström (1988) consider the design problem using stochastic disturbances; Morari and Zafiriou (1989) present a feedforward controller based on IMC parameterization; Söderström (1999) extends the pervious work to correlated disturbances, and in the last years Grimble (1999) proposes an optimal solution for an $\mathcal{H}_\infty/\mathcal{H}_2$ feedforward stochastic control problem.

The classic feedforward controller design frequently suffers from several inherent weaknesses: (i) it requires the identification of the disturbance, and a very good model of the process, something quite difficult for many systems in the chemical industry; (ii) the changes in the process parameters cannot be compensated unless a reliable estimation procedure is incorporated; (iii) it leads to improper transfer functions, so that important simplifications must be done to obtain realizable results.

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This work attempts to partially address the first two problems by proposing a design procedure where process model uncertainties are included. The third point is analyzed using internal-model control (IMC) concepts: the philosophy of the IMC open-loop controller design is used for extending the classical feedforward controller design, this provides a rational procedure to obtain a realizable controller transfer function. Then, the individual feedforward controller and the combined feedforward–feedback control system are tuned using perturbed plant models with known uncertainty limits. The desired closed-loop characteristics for the output and the manipulated variable are handled through suitable objective functions where the feedforward *residual disturbance* plays a key role.

This paper is organized as follows. Section 2 gives the preliminary concepts related to the combined feedforward–feedback control system and introduces the *residual-disturbance* formula. This section exposes also new expressions for nominal and robust performance conditions for disturbance rejection when using the combined control system, and sets the bases for successive or simultaneous tuning. Section 3 proposes a sequence of steps heading to optimum $\mathcal{H}_2$ model-based feedforward controller design, mostly following Morari and Zafiriou (1989). A short analysis of few extraordinary conditions for perfect disturbance rejection is also presented in this section. Section 4 tells how to solve the combined tuning problem as well as the use of linear fractional transformation (LFT) to include additional features like a weight on the manipulated variable. Section 5 presents two application examples that reveal the ability of this approach for tuning both the feedforward and the associated feedback-loop. Finally, the conclusions are presented in Section 6.

2. Properties of the feedforward–feedback control

2.1. Basic relationships

Fig. 1 shows an sketch of the combined control system where $K(s)$ is the feedback controller, $C_f(s)$ is the feedforward controller, $G_d(s)$ stands for the transfer function between the output $y(s)$ and the exogenous disturbance $d(s)$, and $G_p(s)$ is the transfer function between the output and the manipulated variable $u(s)$. From this representation, it is clear that $G_d(s)$ and $G_p(s)$ are linear time invariants (LTI) models representing the real plant.

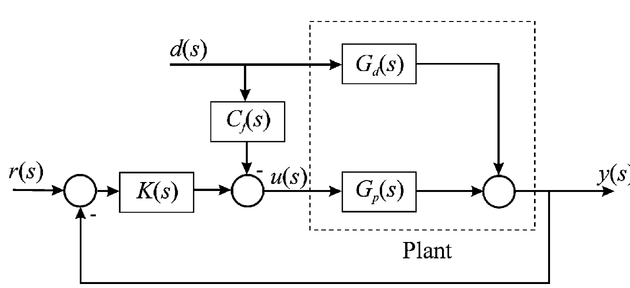

Fig. 1. Feedforward–feedback control system.

The objective for the feedforward controller is basically to generate anticipated corrective actions to compensate measured input disturbances. Standard textbooks like Seborg, Edgar, and Mellichamp (1989) and Stephanopoulos (1984) indicate that a controller $C_f(s)$ connected as shown in Fig. 1, is capable of achieving such goal.

When the feedforward controller is implemented alone, the disturbance effect on the output variable is written as

$$y(s) = [G_d(s) - C_f(s)G_p(s)]d(s) \tag{1}$$

but in case that a feedback-loop is also used, the correct expression is

$$y(s) = \frac{G_d(s) - C_f(s)G_p(s)}{1 + G_p(s)K(s)}d(s) \tag{2}$$

Thus, the ideal feedforward controller that yields $y(s) = 0$ even if $d(s) \neq 0$, is

$$C_f(s) = \frac{G_d(s)}{G_p(s)}. \tag{3}$$

However, the expression (3) might result in an improper or unstable transfer function. When this happens, $C_f(s)$ should be chosen such to minimize the effect of the disturbance on the controlled variable. From inspection of Eq. (2), Morari and Zafiriou (1989) point out that the optimal selection depends on the feedback controller and the disturbance characteristics. They also propose an $\mathcal{H}_2$ optimal feedforward controller design under the theoretical framework supporting the IMC parameterization. Though the proposal accounts for realizability and instability, there are no suggestions about a convenient approach for synthesizing the feedforward controller when there is information about model uncertainties.

2.2. Internal stability

Let us assume inputs $d(t)$ and $r(t)$ are bounded signals, i.e., $d(t), r(t) \in \mathcal{L}_2[0,\infty)$, where $\mathcal{L}_2[0,\infty)$ stands for any continuous signals on $[0,\infty)$ that have finite 2-norm (Green & Limebeer, 1995).

Definition 1. (Perfect disturbance rejection). Feedforward control allows perfect disturbance rejection when the controlled variable does not change as a consequence of any bounded input disturbance, i.e.,

$$y(t) = 0, \quad \forall t \geq 0 \quad \text{for } d(t) \in \mathcal{L}_2[0,\infty) \quad \text{and} \quad r(t) = 0 \tag{4}$$

where $y(t)$ is defined as a deviation from the steady-state value.

Recognizing that $\mathcal{RH}_\infty$ is the real rational subspace of $\mathcal{H}_\infty$ consisting of all proper and rational stable transfer functions (Zhou & Doyle, 1998), a convenient remark rises from the above definition and Eqs. (1)–(3).

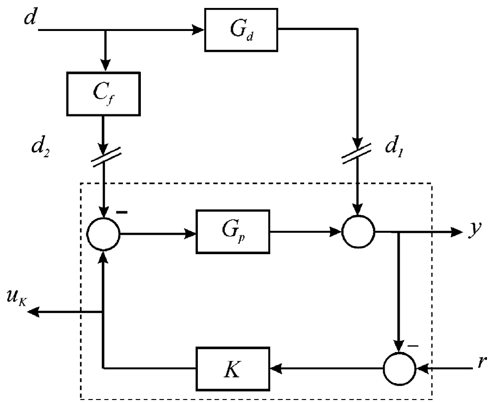

Fig. 2. Internal stability of the feedforward–feedback control system.

Remark 1. (Perfect disturbances rejection). Let us assume there is no mismatch between the actual plant and the LTI nominal models. Let us assume also that the disturbance $d(t)$ is bounded. Then, perfect disturbance rejection is achieved using feedforward controller (3) if this controller belongs to the subspace $\mathcal{RH}_\infty$.

The above statements emphasize the fact that feedforward control allows the possibility of perfect disturbance rejection, something that is not possible using feedback control (Morari & Zafiriou, 1989). Since this property causes the feedforward control to be an interesting control strategy, this work objective is to analyze its strength under realistic cases where model uncertainties are present.

Fig. 2 shows a rearranged sketch of the block diagram in Fig. 1, which serves to analyze the internal stability of the combined feedforward–feedback control system. In this figure, two disturbances affecting the feedback-loop—d_1 and d_2—are identified as internally generated signals. Recalling the definitions of sensitivity function, $S := 1/(1 + KG_\mathrm{p})$, and complementary sensitivity function, $T := KG_\mathrm{p}S$, the internal stability condition can be stated as follows.

Definition 2. (Internal stability of the combined control system). The system shown in Fig. 2 is internally stable if the transfer function matrix

$$M = \begin{bmatrix} (G_\mathrm{d} - C_\mathrm{f}G_\mathrm{p})S & T \\ -K(G_\mathrm{d} - C_\mathrm{f}G_\mathrm{p})S & KS \\ G_\mathrm{d} & 0 \\ C_\mathrm{f} & 0 \end{bmatrix} \tag{5}$$

from $[d\ r]^\mathrm{T}$ to $[y\ u_k\ d_1\ d_2]^\mathrm{T}$, belongs to $\mathcal{RH}_\infty$ for all bounded $d(t)$ and $r(t)$.

Note that requiring internal stability to the feedback-loop alone is not sufficient for the combined system. For combined internal stability it is also necessary that $C_\mathrm{f}(s)$ and $G_\mathrm{d}(s)$ be stable functions. Otherwise, for any $d(t) \in \mathcal{L}_2[0, \infty)$, neither $d_1(t)$ nor $d_2(t)$ will be bounded functions. In other words, if the controller $C_\mathrm{f}(s)$ and the plant $G_\mathrm{d}(s)$ belong to the subspace $\mathcal{RH}_\infty$, the internal stability of the feedback-loop is not affected since all the input signals are bounded, and there is no effect on the characteristic equation.

Remark 2. The combined feedforward–feedback control system is internally stable if $G_\mathrm{d}(s)$ and $C_\mathrm{f}(s)$ belong to $\mathcal{RH}_\infty$ and $K(s)$ is an internally stabilizing feedback controller.

2.3. Nominal and robust stability

To extend the analysis to realistic cases, let us assume that there are available two nominal models and the corresponding uncertainty limits to represent the real plant. Hence, the entire plant dynamics can be described by means of two perturbed models, which are defined using the classic multiplicative uncertainty, namely

$$G_\mathrm{p}(s) = G_{\mathrm{p_o}}(s)(1 + lm_\mathrm{u}(s)), \tag{6}$$

$$G_\mathrm{d}(s) = G_{\mathrm{d_o}}(s)(1 + lm_\mathrm{d}(s)), \tag{7}$$

where $G_{\mathrm{p_o}}$ and $G_{\mathrm{d_o}}$ are the nominal models. Two families of models Π_d and Π_u are involved here, explicitly

$$\Pi_\mathrm{u} := \left\{ G_\mathrm{p} : |lm_\mathrm{u}(\mathrm{j}\omega)| := \left| \frac{G_\mathrm{p}(\mathrm{j}\omega) - G_{\mathrm{p_o}}(\mathrm{j}\omega)}{G_{\mathrm{p_o}}(\mathrm{j}\omega)} \right| \le \overline{lm}_\mathrm{u}(\omega) \right\}, \tag{8}$$

$$\Pi_\mathrm{d} := \left\{ G_\mathrm{d} : |lm_\mathrm{d}(\mathrm{j}\omega)| := \left| \frac{G_\mathrm{d}(\mathrm{j}\omega) - G_{\mathrm{d_o}}(\mathrm{j}\omega)}{G_{\mathrm{d_o}}(\mathrm{j}\omega)} \right| \le \overline{lm}_\mathrm{d}(\omega) \right\}, \tag{9}$$

where $\overline{lm}_\mathrm{u}(\omega)$ and $\overline{lm}_\mathrm{d}(\omega)$ are the upper bounds for the multiplicative uncertainty modulus $|lm_\mathrm{u}(\mathrm{j}\omega)|$ and $|lm_\mathrm{d}(\mathrm{j}\omega)|$, respectively. Note that, due to the internal stability condition, all members of Π_d must belong to $\mathcal{RH}_\infty$.

Then, the Remark 2 and the expressions (6)–(9) allow the following definitions.

Definition 3. (Nominal stability of the combined control system). The feedforward–feedback control system have nominal stability if the feedback controller $K(s)$ gives internal stability to the closed-loop with the nominal model $G_{\mathrm{p_o}}(s)$ and, $G_{\mathrm{d_o}}(s)$ and $C_\mathrm{f}(s)$ belong to $\mathcal{RH}_\infty$.

Definition 4. (Robust stability of the combined control system). The feedforward–feedback control system have robust stability if the feedback controller $K(s)$ gives internal stability to the closed-loop for all $G_\mathrm{p}(s) \in \Pi_\mathrm{u}$, and $C_\mathrm{f}(s)$ and all $G_\mathrm{d}(s) \in \Pi_\mathrm{d}$ belong to $\mathcal{RH}_\infty$.

2.4. Nominal and robust performance

A main concern in this work is the performance of the combined control system. To address this problem let us substitute the sensitivity function $S := 1/(1+KG_p)$, and the perturbed models (6) and (7) in the Eq. (2). This gives

$$\frac{y(s)}{d(s)} = S(s)\left\{G_{d_o}(s)\,[1+lm_d(s)] - C_f(s)G_{p_o}(s)\,[1+lm_u(s)]\right\}. \quad (10)$$

The right side of this expression suggests the definition of a new transfer function as

$$R_d(s) := G_{d_o}(s)\,[1+lm_d(s)] - C_f(s)G_{p_o}(s)\,[1+lm_u(s)], \quad (11)$$

which can be thought as the *open-loop residual disturbance* remaining after the feedforward compensation. Hence, Eq. (10) can be rewritten as

$$y(s) = S(s)R_d(s)d(s), \quad (12)$$

from where it is readily seen that the condition

$$|S(j\omega)R_d(j\omega)| \le \gamma, \quad \forall\, 0 \le \omega < \infty, \quad (13)$$

ensures

$$\|y(j\omega)\| \le \gamma\,\|d(j\omega)\|. \quad (14)$$

It is also apparent from (14), that the disturbance attenuation of the output variable is achieved if $\gamma < 1$. This condition however, refers to the inequality (13), which links the residual disturbance to the feedback sensitivity function, and leads to the following statements: (i) the fine tuning of a feedforward controller has to be done considering the feedback-loop characteristics or; (ii) both, feedforward and feedback controllers should be simultaneously tuned for an efficient disturbance rejection.

To formalize this result, recall that if the performance objectives are satisfied for the nominal plant, the control system reaches what is named nominal performance.

Definition 5. (Nominal performance for disturbance rejection). The combined feedforward–feedback control system attains nominal performance if, for every $d(t) \in \mathcal{L}_2[0,\infty)$, the desired performance objectives are satisfied for the nominal models G_{p_o} and G_{d_o}.

Let $\|\cdot\|_\infty$ denote the $\mathcal{H}$ infinity norm based on the classical definition, also let S_0 stand for the nominal sensitivity function $S_0 := 1/(1+KG_{p_o})$, and the *nominal residual disturbance* be $R_{d_o} := G_{d_o} - C_f G_{p_o}$. Then, the performance objective can be expressed by the following lemma.

Lemma 1. (*Nominal performance for disturbance rejection*). *The feedforward–feedback control system reaches the nominal performance for disturbance rejection if*

$$\left\|w_p\, S_0\, R_{d_o}\right\|_\infty \le 1 \quad (15)$$

is satisfied, where w_p is a weight function included for a more general formulation.

Letting $d(t) \in \mathcal{L}_2[0,\infty)$, the proof is trivial since the inequality (13) can be rewritten for nominal models as Eq. (15), where $\gamma^{-1} = |w_p| \ge 1$.

Furthermore, the expression (13) represents the robust performance condition for the combined feedforward–feedback control system when, being $\gamma < 1$, it holds for all the models included in the families Π_u and Π_d.

Definition 6. (Robust performance for disturbance rejection). The combined feedforward–feedback control system attains robust performance if, for every $d(t) \in \mathcal{L}_2[0,\infty)$, the desired performance condition is satisfied for every perturbed model $G_p \in \Pi_u$ and $G_d \in \Pi_d$.

Similarly to Lemma 1, this sufficient condition can be formalized as follows:

$$\left\|w_p\, S\, R_d\right\|_\infty \le 1 \quad (16)$$

Lemma 2. (*Robust performance of the combined control system*). *Let $G_p(s)$ and $G_d(s)$ belong to the families Π_u and Π_d, respectively, then the combined feedforward–feedback control system achieves robust performance if the following condition is satisfied*:

$$|w_p(j\omega)S_0(j\omega)R_d(j\omega)| + |T_0(j\omega)lm_u(j\omega)| \le 1, \quad \forall\, 0 \le \omega < \infty. \quad (17)$$

where $T_0 = 1 - S_0$.

The proof of Lemma 2 is included in Appendix A. Relevant result arises from this analysis by noting that the first term in Eq. (17) does not completely match the condition (15) for Nominal Performance, and that $\|T_0 lm_u\|_\infty \le 1$ represents the necessary and sufficient condition for robust stability of the feedback-loop. The point is that in robust control theory, it is frequently accepted that nominal performance plus robust stability implies robust performance, but according to the Eq. (17) this is not the case for the combined feedforward–feedback control system where R_d is different form the nominal R_{d_o} in Eq. (15). Furthermore, it is worth to note that Eq. (17) reduces to the traditional feedback condition in the absence of feedforward action.

3. Feedforward controller design

This section discusses the development of a design procedure for feedforward controllers when the process model have reliability problems, i.e., when it is not possible to neglect the effect of model uncertainties on the control performance, or to ignore that the actual plant dynamics are significantly more complex than the assumed linear model.

The procedure proposed here for the synthesis of a dynamic feedforward controller consists of determining

$$C_{f_o}(s) = \Gamma_1\left\{\frac{G_{d_o}(s)}{G_{p_o}(s)}\right\}, \quad (18)$$

where $G_{d_o}(s)$ and $G_{p_o}(s)$ are the nominal transfer functions, and $\Gamma_1\{\cdot\}$ is an operator that excludes the following parts from the polynomial ratio: (i) the zeros of $G_{p_o}(s)$ that belong to the right-half complex plane; (ii) the unstable poles of $G_{p_o}(s)$; and (iii) the positive time delay that might result from the difference $\theta_{d_o} - \theta_{p_o}$, where θ_{d_o} and θ_{p_o} are the nominal delays in $G_{d_o}(s)$ and $G_{p_o}(s)$, respectively.

Though the operation defined in (18) gives a stable transfer function, in many cases the result is no realizable. Therefore, an auxiliary filter function is proposed to solve the realizability problem. The final controller expression is then given by

$$C_f(s) = C_{f_o}(s) f(s), \tag{19}$$

where *f*(*s*) is defined as follows.

Definition 7. (Feedforward filter). The feedforward filter *f*(*s*) is chosen as a stable rational transfer function,

$$f(s) := \frac{K_f}{(\lambda_f s + 1)^n}, \tag{20}$$

where λ_f and K_f are positive tuning parameters and *n* is determined by

$$n := \begin{cases} |\Gamma_2| & \text{if} \quad \Gamma_2\{C_{f_o}(s)\} < 0 \\ 0 & \text{if} \quad \Gamma_2\{C_{f_o}(s)\} \geq 0 \end{cases} \tag{21}$$

In this expression $\Gamma_2\{\cdot\}$ stands for an operator that computes the relative order of $C_{f_o}(s)$, in other words, it gives the difference between denominator and numerator order.

Notice that in case the complete relationship $\{G_{d_o}/G_{p_o}\}$ is included in the design (no zero or pole is excluded from G_{p_o} or G_{d_o}) and C_{f_o} is realizable, the feedforward controller leads to perfect disturbance rejection only if the model uncertainties are negligible. This statement is supported by a short analysis of the residual disturbance R_d, which can be written

$$R_d(s) = G_{d_o}(s)[1 + lm_u(s)] \times \left\{ \frac{[1 + lm_d(s)]}{[1 + lm_u(s)]} - C_f(s)\frac{G_{p_o}(s)}{G_{d_o}(s)} \right\}. \tag{22}$$

Assuming the particular case in which the operator Γ_1 does not exclude any term, the full cancellation of G_{p_o} and G_{d_o} in Eq. (22) yields

$$R_d(s) = G_{d_o}(s)[1 + lm_u(s)] \left\{ \frac{[1 + lm_d(s)]}{[1 + lm_u(s)]} - f(s) \right\}. \tag{23}$$

It is apparent from expression (23) that if C_{f_o} is realizable and both multiplicative uncertainty are negligible, namely, $lm_d(s) \cong 0$ and $lm_u(s) \cong 0$, then $f(s) = K_f = 1$ would be enough to ensure disturbance rejection since $R_d \cong 0$. However, if the uncertainties are not negligible and $f(s) = [1 + lm_d(s)]/[1 + lm_u(s)]$ is adopted, the residual disturbance will rarely remain null all the time. This happens because the actual concept underlying the uncertainty functions lm_d and lm_u is that they do not describe deterministic dynamics such to make possible an exact compensation. Global model uncertainties like these are useful functions for providing limit behaviors only.

4. Tuning procedures

4.1. Tuning the feedforward controller

Most feedforward controllers designed as indicated before have the filter gain K_f and the filter time constant λ_f as adjusting parameters. For a robust performance tuning, the following frequency domain problem can be proposed: find K_f and λ_f such that

$$\left\| w_p S R_d \right\|_\infty = 1, \tag{24}$$

where w_p stands for a weighting function that provides an additional degree of freedom for defining the output-response characteristics. Since the tuning parameters, K_f and λ_f, belong to the residual disturbance R_d exclusively, a question might arise in regard to the possibility of using R_d alone. However, the arguments to include the feedback sensitivity function *S* in (24) were explained in Section 2, based on the convenience of referring the disturbance-rejection problem to the expression in Eq. (13). Given a feedback controller previously tuned, the feedforward solution to the above problem considers the frequency-spectrum characteristics of that specific feedback-loop. Alternatively, the use of R_d alone would give a feedforward-controller tuning independent from the feedback-loop, and probably, of lower combined performance.

Observe also that (24) defines a loop-shaping procedure rather than an optimization problem since it is equivalent to the condition (13), where $\gamma = |w_p|^{-1} \leq 1$. This strategy is quite different, for instance, from the problem of minimizing $\left\| w_p S R_d \right\|_\infty$ where the non-linear terms lead to a result that must not be considered an optimum since the global optimality is not guaranteed. Even more important: this pseudo-optimal search might attempt the ideal $\lambda_f = 0$ solution, which is unrealizable.

4.2. Tuning the feedforward–feedback control system

Recall that the feedback controllers are usually tuned to satisfy one of the following two objectives: (i) good set-point tracking; or (ii) good disturbance rejection. In this subsection, the combined feedforward–feedback tuning for the regulation problem is analyzed.

The simplest strategy is to adjust the feedforward controller such to reduce the effect of the disturbance in the presence of an specific feedback controller and simultaneously to tune the feedback controller such to complete the disturbance rejection to the level indicated by w_p. Although the tuning problem in Eq. (24) can be extended to include the feedback controller by simply allowing the variation of

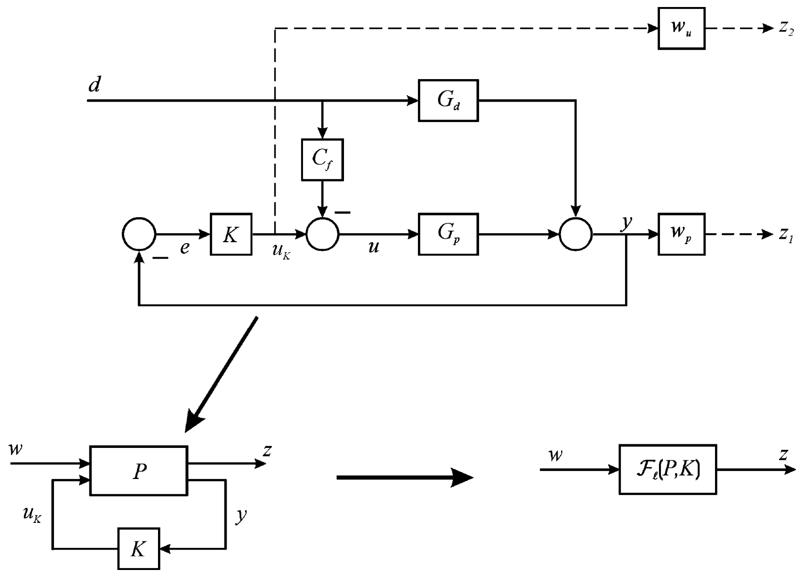

Fig. 3. Linear fractional transformation of the feedforward–feedback control system. Here, $w = d$, $z = [z_1\ z_2]^{\mathrm{T}}$, $\mathcal{F}_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$ with $P_{11} = [w_{\mathrm{p}}R_{\mathrm{d}}\ 0]^{\mathrm{T}}$, $P_{12} = [w_{\mathrm{p}}G_{\mathrm{p}}\ w_{\mathrm{u}}I]$, $P_{21} = R_{\mathrm{d}}$ and $P_{22} = G_{\mathrm{p}}$.

additional parameters—like the proportional gain and the integral-reset time in case that a PI controller is used—an alternative functional can be defined according to the needs.

For instance, following Zhou and Doyle (1998) or Sanchez Peña and Sznaier (1998), the linear fractional transformation (LFT) can be applied to the combined feedforward–feedback control system, as indicated in Fig. 3. Hence, attending to the new definition of the system output, the following mixed functional can be proposed which includes performance limits for the manipulated variable,

$$N = \begin{bmatrix} w_{\mathrm{p}}SR_{\mathrm{d}} \\ -w_{\mathrm{u}}(K\,SR_{\mathrm{d}}) \end{bmatrix} \tag{25}$$

where, w_{u} and w_{p} are weight functions used to restrict the actuator movements and to determine the performance required to the combined control system respectively. This way, the index function $w_{\mathrm{p}}SR_{\mathrm{d}}$ used in (24) is mixed with another term that focus on the intensity of the manipulated variable movements. The extension from the performance index (24) to a new one using (25) revels that the combined system can also be analyzed and tuned under different $\mathcal{H}_\infty$ performance requirements.

Remark 3. The general combined feedforward–feedback tuning for the regulation problem can be presented as an $\mathcal{H}_\infty$ mixed criteria problem, namely

$$\|N\|_\infty \le 1. \tag{26}$$

where N is a functional defining the required performance and where the solution is constrained to parameter spaces ensuring a stable control system.

Besides, note that when the feedback-loop is used alone to solve the regulation problem, the mixed criterion in Eq. (25) changes to

$$N = \begin{bmatrix} w_{\mathrm{p}}SG_{\mathrm{d}} \\ -w_{\mathrm{u}}KSG_{\mathrm{d}} \end{bmatrix}, \tag{27}$$

where N now includes the transfer function G_{d} defining the disturbance characteristics instead of the residual disturbance R_{d}. The main difference between these two functions is that R_{d} includes adjusting parameters in contrast with G_{d}. Hence, a valuable insight about the effect of adding a feedforward controller to a feedback control loop is obtained by comparing Eqs. (25) and (27).

Remark 4. A functional defining regulation criteria for the combined feedforward–feedback tuning is structurally similar to the functional defining the same criteria for the feedback-loop alone, with the only difference that the residual disturbance transfer function $R_{\mathrm{d}}(s)$ must be used in place of the original disturbance transfer function $G_{\mathrm{d}}(s)$.

5. Application examples

5.1. Example 1

Consider the design of a feedforward–feedback control system like sketched in Fig. 1, where

$$G_{\mathrm{d}}(s) = \frac{K_{\mathrm{d}}\mathrm{e}^{-\theta_{\mathrm{d}}s}}{(\tau_{\mathrm{d}}s+1)} \quad \text{and} \quad G_{\mathrm{p}}(s) = \frac{K_{\mathrm{p}}\mathrm{e}^{-\theta_{\mathrm{p}}s}}{(\tau_{\mathrm{p1}}s+1)(\tau_{\mathrm{p2}}s+1)}. \tag{28}$$

In this problem, $[K_{\mathrm{d}}, \tau_{\mathrm{d}}, \theta_{\mathrm{d}}]$ and $[K_{\mathrm{p}}, \tau_{\mathrm{p1}}, \tau_{\mathrm{p2}}, \theta_{\mathrm{p}}]$ are uncertain parameters of $G_{\mathrm{d}}(s)$ and $G_{\mathrm{p}}(s)$, respectively. Let us adopt as nominal parameter values $K_{\mathrm{d_o}} = 1$, $\tau_{\mathrm{d_o}} = 1$, $\theta_{\mathrm{d_o}} =$

Table 1
Limit values for the uncertain parameters in Example 1

	Minimum	Maximum
K_d	0.8	1.2
τ_d	0.9	1.1
θ_d	0.095	0.105
K_p	0.95	1.05
τ_{p1}	0.95	1.05
τ_{p2}	0.95	1.05
θ_p	0.19	0.21

0.1, $K_{p_o} = 1$, $\tau_{p1_o} = 1$, $\tau_{p2_o} = 1$, and $\theta_{p_o} = 0.2$, with uncertainty ranges given by Table 1. Assume also, that the feedback regulator is a PID controller with settings determined by (Morari & Zafiriou, 1989),

$$K_C = \frac{\tau_{p1_o} + \tau_{p2_o}}{K_{p_o}(\varepsilon + \theta_{p_o})},$$

$$T_I = \tau_{p1_o} + \tau_{p2_o} \quad \text{and} \quad T_D = \frac{\tau_{p1_o}\tau_{p2_o}}{\tau_{p1_o} + \tau_{p2_o}} \tag{29}$$

Furthermore, according to Eqs. (18)–(21), the design expression for the feedforward controller is

$$C_f(s) = \frac{K_{d_o} K_f}{K_{p_o}} \frac{(\tau_{p1_o}s + 1)(\tau_{p2_o}s + 1)}{(\tau_{d_o}s + 1)(\lambda_f s + 1)}. \tag{30}$$

The parameter intervals in Table 1 define convex hulls in the parameters space that are projected on the complex plane, generating uncertainty regions like shown in Fig. 4. The maximum bounds $\overline{lm}_u$ and $\overline{lm}_d$ are numerically determined as the largest distance between the nominal model and the respective family boundary.

The following tuning approaches are tested: (1) the combined feedback–feedforward settings are determined by

Table 2
Settings obtained for different limit values of γ

γ	ε	K_f	λ_f
1.00	2.93	2.29	0.0256
0.75	3.51	1.97	0.0242
0.50	3.98	1.67	0.0244
0.25	4.60	1.39	0.0233
0.15	4.88	1.10	0.0127

solving the problem (24) for different constant values of w_p; (2) the mixed performance index in Eq. (25) is used to observe the effect of changing the weight w_u on the output performance.

The numerical solutions to these tuning problems did not show sensibility to the initial parameter values even though they include nonlinear relationships. However, a simple but reasonable initialization was used: $\varepsilon = 1$ for being close to fast closed-loop response conditions, $K_f = 1$ and, a small value of λ_f for being not too far from ideal compensation.

In the first tuning approach, both the feedforward and feedback controller parameters are determined simultaneously using (24). The problem basically consists on finding the set of parameters $\{\varepsilon, K_f, \lambda_f\}$ satisfying an arbitrary value of $\gamma = |w_p|^{-1} \leq 1$. The numerical experience is run here for five different upper limits, like shown in Fig. 5 where $|SR_d|$ is plotted versus frequency. The settings obtained in each case are exposed in Table 2. Figs. 6 and 7 show the time responses of the system output and the control variable, respectively, obtained for a unit-step change in the disturbance. Clearly, the disturbance rejection improves as the upper limit decreases.

Fig. 8 shows four frequency responses under different control conditions. The first curve corresponds to $|y(j\omega)|$

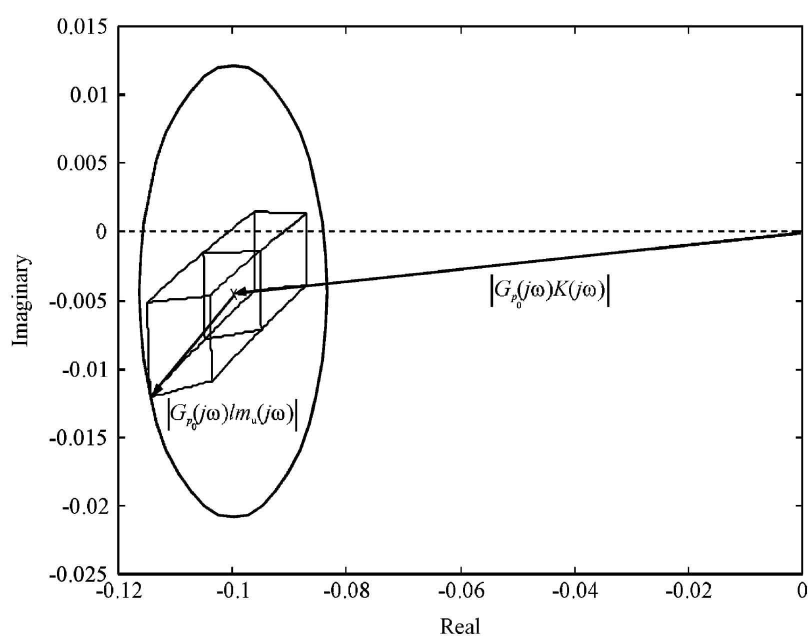

Fig. 4. Maximum uncertainty bound $\overline{lm}_u$ computed for a given frequency ω.

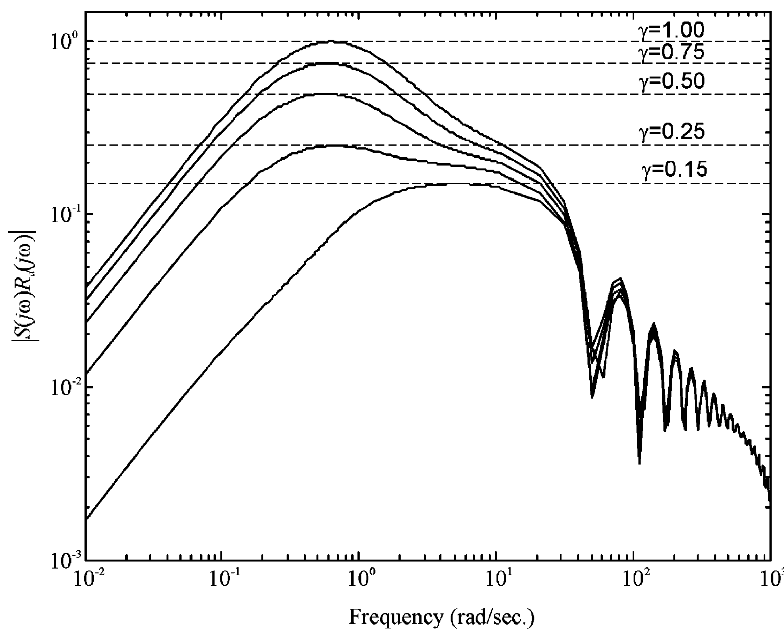

Fig. 5. Simultaneous tuning of the combined control system for different γ values.

$= |G_d(j\omega)|$, which obviously shows the complete disturbance effect on the output when the system is not controlled at all. The second curve is the result of incorporating the feedback controller only, namely, this is the plot of $|y(j\omega)| = |S(j\omega)G_d(j\omega)|$ using $\varepsilon = 4.88$, the same feedback tuning shown in Table 2 for $\gamma = 0.15$. The third one, is the open-loop residual disturbance $|y(j\omega)| = |R_d(j\omega)|$ showing the important rejection obtained by the feedforward controller alone; the settings in this case are $K_f = 1.10$ and $\lambda_f = 0.0127$. Finally, the last curve shows the effect of combining both controllers, which, in fact, were tuned together to satisfy the upper limit $\gamma = |w_p|^{-1} = 0.15$.

Table 3
Settings obtained for different values of w_u and constant $\gamma = 1$

w_u	ε	K_f	λ_f
1	3.27	1.36	0.024
10	33.8	1.23	0.036
15	50.0	1.18	0.13
20	66.6	1.15	0.22

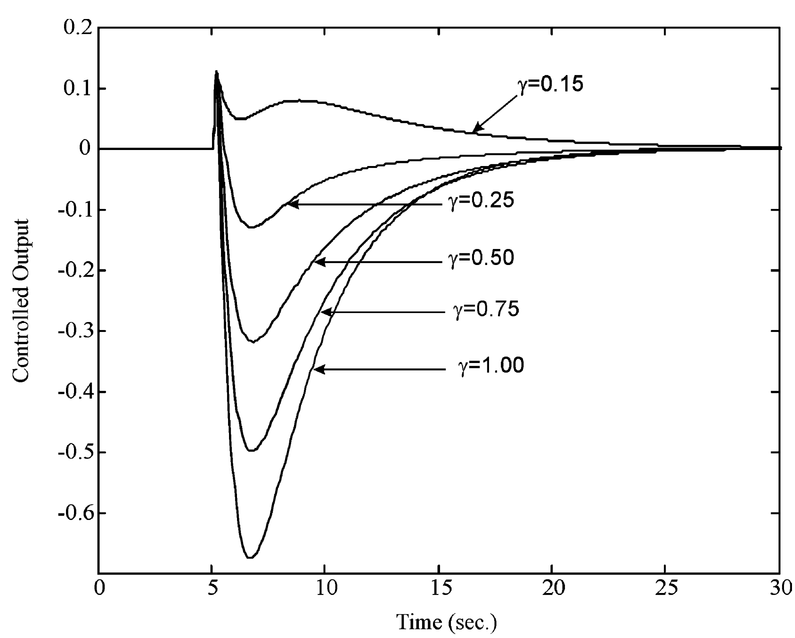

Fig. 6. Output-variable responses to a unit-step change in the disturbance using settings in Table 2.

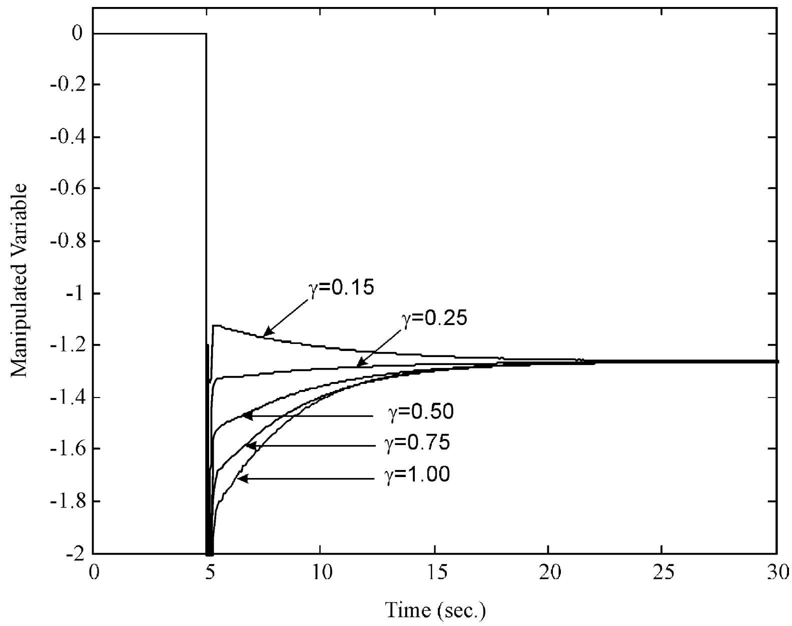

Fig. 7. Manipulated variable movements that correspond to the responses in Fig. 6.

This analysis shows the role played by each controller to achieve the disturbance-rejection objective. Note that both, the contribution made by the feedforward controller without endangering the stability, and the feedback influence at low frequencies can be simultaneously conditioned by the weighting function w_p.

In the second tuning approach used in this application example, the combined control system is tuned using the mixed performance index (25) for $w_p = 1$, and different values of the weight w_u. The settings obtained in this case are given in Table 3, while Figs. 9 and 10 show the corresponding output and manipulated time response. The initial manipulated-variable stroke is important in all cases (-56.2; -33.9; -9.12; -5.2), with the larger one being that correspondent to the most demanding settings ($w_u = 1$).

5.2. Example 2

In this second example, the plant shows inverse responses to changes in the manipulated variable, which delays the feedback control action and makes the control problem more

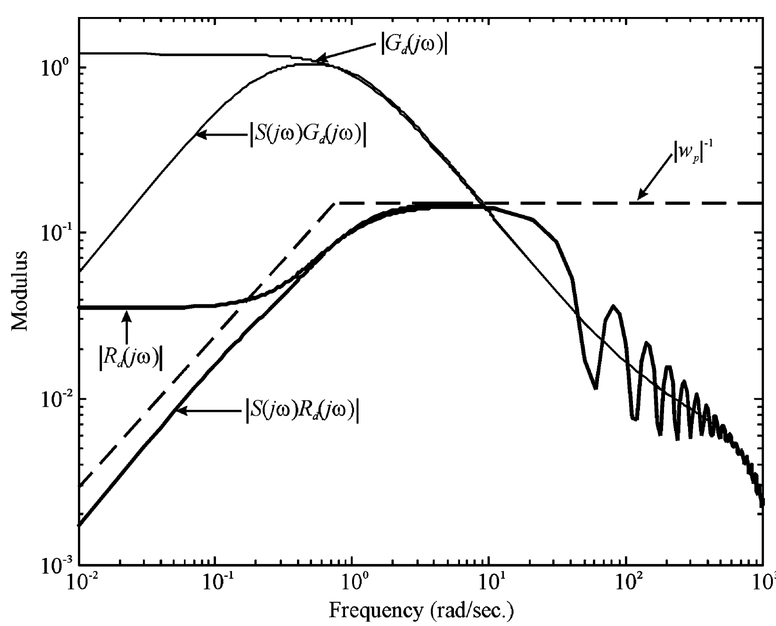

Fig. 8. Effect of using feedback and feedforward controllers separately, and the combined feedforward–feedback control system.

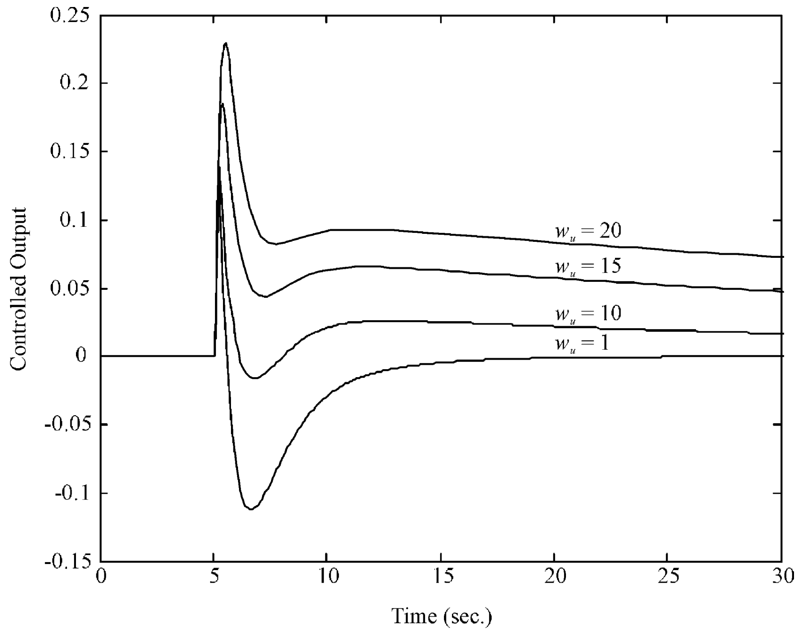

Fig. 9. Output-variable responses to a unit-step change in the disturbance and different weights w_u.

challenging. Thus, the plant is modeled in this case by

$$G_d(s) = \frac{K_d e^{-\theta_d s}}{\tau_d s + 1}, \quad \text{and} \quad G_p(s) = \frac{K_p(-z_p s + 1)e^{-\theta_p s}}{\tau_p^2 s^2 + 2\zeta\tau_p s + 1}, \tag{30}$$

and the feedforward controller takes the form

$$C_f(s) = \frac{K_d K_f}{K_p} \frac{(\tau_p^2 s^2 + 2\xi\tau_p s + 1)}{(\tau_d s + 1)(\lambda_f s + 1)}. \tag{31}$$

Let us adopt the following parameter values $K_d = 1$, $\theta_d = 1$, $\theta_d = 0.5$, $K_p = 1$, $\tau_p = 2.0$, $z_p = 2.0$, $\xi = 0.23$, and $\theta_p = 1.5$. The model uncertainties are assumed to depend exclusively on the time delays; this is equivalent to say that the multiplicative uncertainties for G_p and G_d are given by $lm_u(s) = e^{-\theta_p s} - 1$, and $lm_u(s) = e^{-\theta_p s} - 1$, respectively.

The strategy used for designing the feedback controller follows again the IMC parameterization given by Morari and Zafiriou (1989); in this case $G_p(s)$ leads to a PID controller

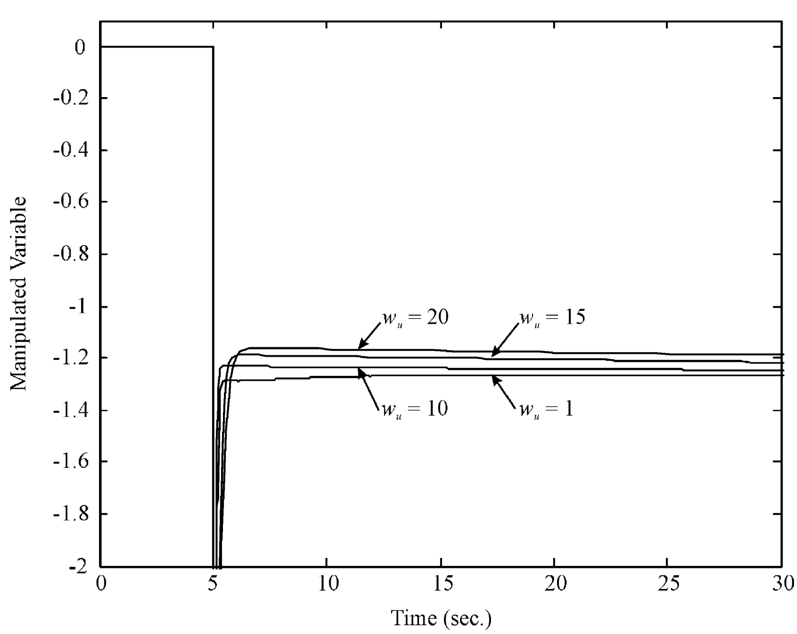

Fig. 10. Manipulated variable movements that correspond to the responses in Fig. 9.

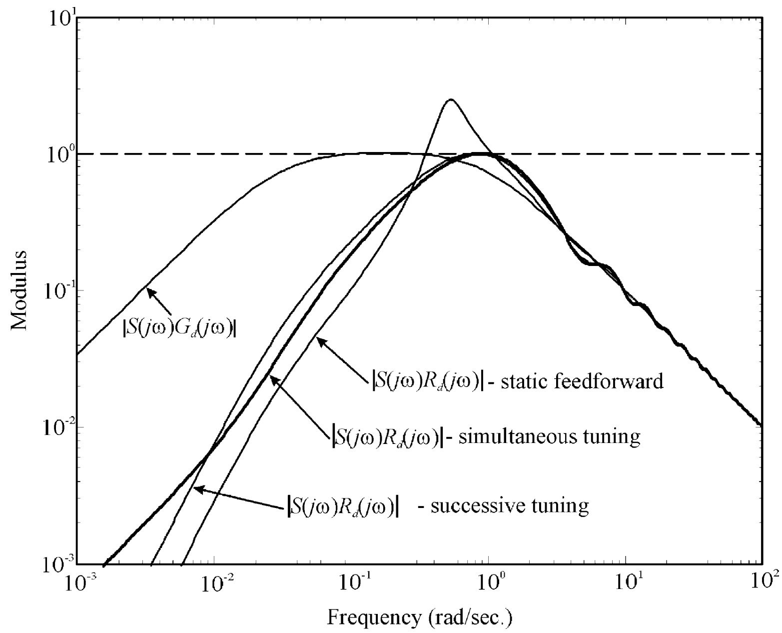

Fig. 11. Second example with $\tau_d = 1.0$. Output-variable modulus for different tuning procedures.

with settings determined by

$$K_C = \frac{2\xi\tau}{K_p(2z_p + \varepsilon)}, \qquad T_I = 2\xi\tau, \quad \text{and} \quad T_D = \frac{\tau}{2\xi}. \quad (32)$$

Successive and simultaneous tunings are tested in this application example. In the successive tuning, the feedback parameter ε is first adjusted to satisfy $|S(j\omega)G_d(j\omega)| = 1$; then, the feedforward parameters K_f and λ_f are adjusted to obtain $|S(j\omega)R_d(j\omega)| = 1$ while maintaining ε constant. In the simultaneous case, the feedforward–feedback tuning is solved using ε, K_f, and λ_f as free parameters to satisfy $|S(j\omega)R_d(j\omega)| = 1$ directly. The settings obtained in these different adjustments are shown in the first part of Table 4.

Fig. 11 shows the index functions versus frequency determined by the settings in Table 4, and Fig. 12 shows the corresponding time-domain responses to a unit-step change in the disturbance. None of these figures shows a significant difference between successive or simultaneous tuning. Surprisingly, no even the simple static feedforward controller shows a large difference in the frequency domain, other than a pick around $\omega = 0.5$–0.6 radians/s; however,

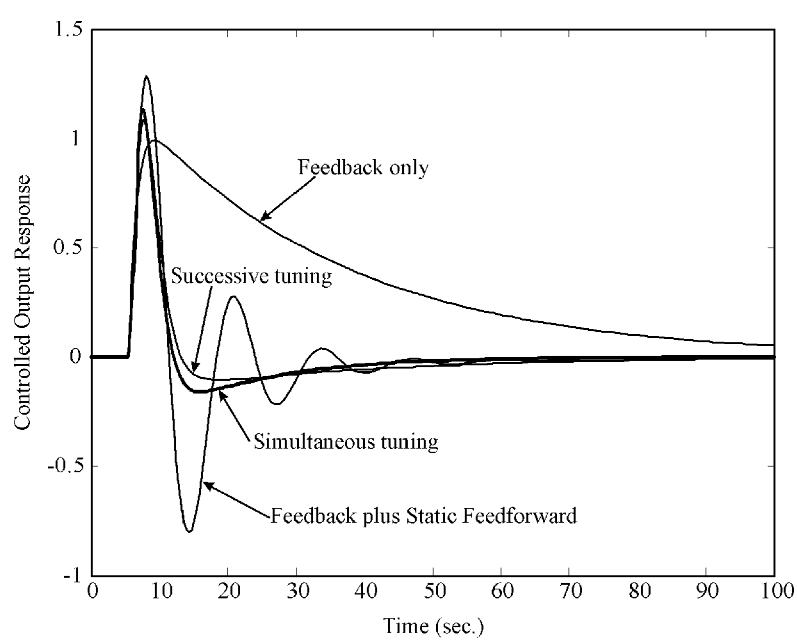

Fig. 12. Time-domain responses that correspond to the results in Fig. 11.

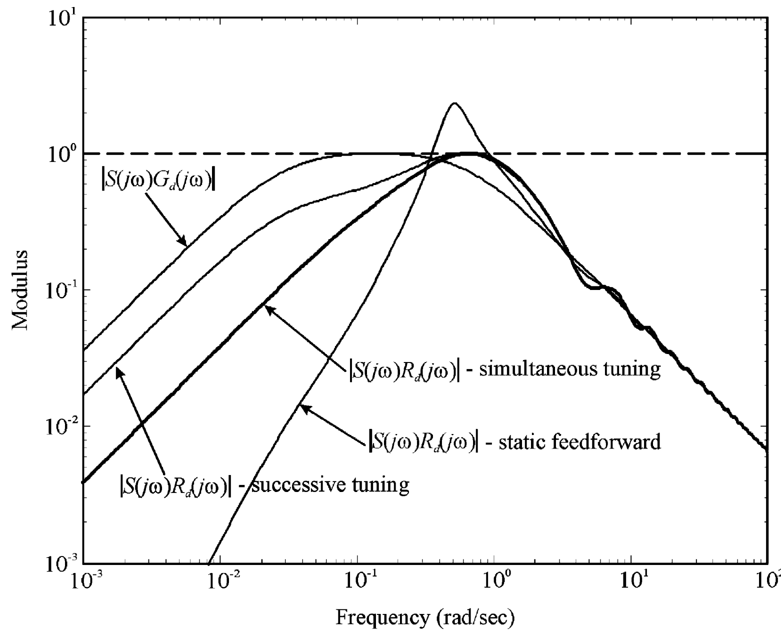

Fig. 13. Second example with $\tau_d = 1.5$. Output-variable modulus for different tuning procedures.

the time-domain response shows the effect of ignoring the $|S(j\omega)R_d(j\omega)| = 1$ condition. A slight difference in favor of the simultaneous tuning can be observed since the time domain response shows a sorter settling time.

Figs. 13 and 14 show the results of similar tuning procedures, but this time changing $\tau_d = 1.0$ by $\tau_d = 1.5$ in $G_d(s)$. An important difference between the successive and the simultaneous tuning is observed in these responses, stressing the convenience of adopting simultaneous tuning as a standard procedure. The settings determined in this opportunity are shown in the second part of Table 4.

Table 4
Settings obtained for Example 2

Tuning procedure	ε	K_f	λ_f
Case $\tau_d = 1.0$			
Successive tuning	30.0	0.997	1.339
Simultaneous tuning	14.62	0.966	1.00
Static feedforward	30.0	$C_{ff} = K_d/K_p = 1.0$	
Case $\tau_d = 1.5$			
Successive tuning	32.0	1.476	1.475
Simultaneous tuning	7.85	1.329	1.200
Static feedforward	32.0	$C_{ff} = K_d/K_p = 1.0$	

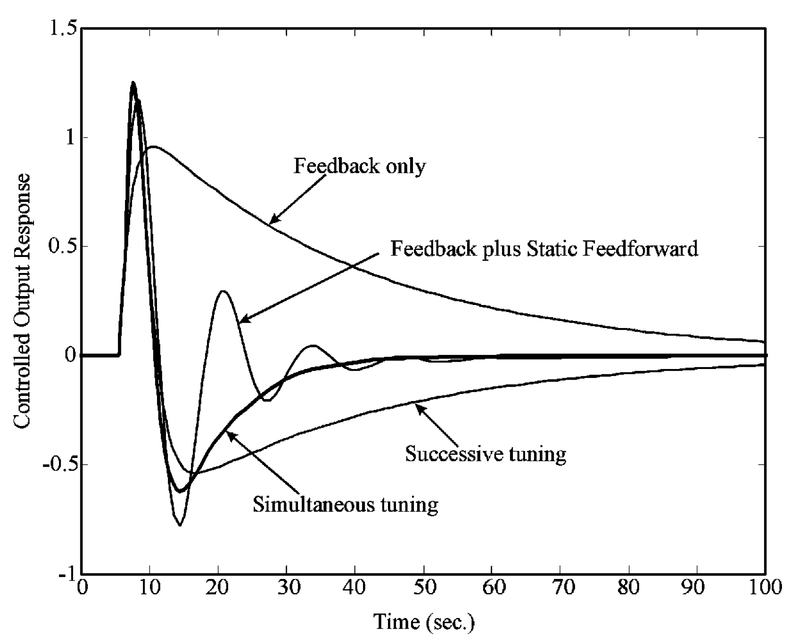

Fig. 14. Time-domain responses that correspond to the results in Fig. 13.

6. Conclusions

This work revises the synthesis of feedforward controllers when the available dynamic models include estimated limits for the uncertainties. The following statements summarize concepts analyzed along this paper, which contribute to solve disturbance-rejection problems.

Though it is well known that feedforward controller does not influence feedback stability, the analysis of *internal stability* for the combined feedforward–feedback control system requires that the feedforward controller be a stable function, and that the disturbance effect on the output be a bounded signal.

The analysis of including the feedforward controller in a control system is facilitated by the definition of the *open-loop residual disturbance*, which represents that part of the original disturbance remaining in the controlled output despite of the feedforward control action. This residual disturbance defines the regulation task for the feedback controller that completes the disturbance rejection.

Typically, nominal performance plus robust stability implies robust performance, however the analysis presented in this paper demonstrates that this is not the case for the combined feedforward–feedback control system.

The feedforward and the feedback controllers should be simultaneously tuned for an efficient disturbance rejection; alternatively, fine-tuning of the feedforward controller must consider the spectral characteristics of the associated feedback-loop.

Further contributions of this work are exposed in Section 2 through a set of definitions, lemmas and remarks that extends classic feedback control concepts to the combined feedforward–feedback control system.

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Appendix A

Proof of Lemma 2. *According to* Eq. (16),

$$|w_{\rm P}(j\omega)S(j\omega)R_{\rm d}(j\omega)| \le 1, \quad \forall\, 0 \le \omega < \infty. \tag{A.1}$$

Defining $L_0 := G_{\rm p_o}K$, then $S = [1 + L_0(1 + lm_{\rm u})]^{-1}$ and,

$$1 + L_0(1 + lm_{\rm u}) = \frac{1}{S_0}[(S_0 + S_0L_0) + S_0L_0lm_{\rm u}] \tag{A.2}$$

Using the complementary sensitivity function $T_0 = S_0L_0$ and recalling that $T_0 + S_0 = 1$ allows to write, $1 + L_0(1 + lm_{\rm u}) = (1 + T_0lm_{\rm u})/S_0$. Replacing this expression in the definition of sensitivity function,

$$S = S_0[1 + T_0lm_{\rm u}]^{-1}, \tag{A.3}$$

and substituting (A.3) into (A.1) gives,

$$|w_{\rm p}(j\omega)S_0(j\omega)[1 + T_0(j\omega)lm_{\rm u}(j\omega)]^{-1}R_{\rm d}(j\omega)| \le 1, \quad \forall\, 0 \le \omega < \infty. \tag{A.4}$$

Rearranging the above condition gives

$$|w_{\rm p}S_0(1 + T_0lm_{\rm u})^{-1}R_{\rm d}| \le |w_{\rm p}S_0R_{\rm d}||(1 + T_0lm_{\rm u})^{-1}| = \frac{|w_{\rm p}S_0R_{\rm d}|}{|1 + T_0lm_{\rm u}|} \le \frac{|w_{\rm p}S_0R_{\rm d}|}{1 - |T_0lm_{\rm u}|}. \tag{A.5}$$

Hence, from (A.4) and the inequality (A.5), the following expression can be obtained,

$$\frac{|w_{\rm p}S_0R_{\rm d}|}{1 - |T_0lm_{\rm u}|} \le 1. \tag{A.6}$$

Rearranging again,

$$|w_{\rm p}S_0R_{\rm d}| + |T_0lm_{\rm u}| \le 1, \quad \forall\, 0 \le \omega < \infty \tag{A.7}$$

which is the result in expression (17). □

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