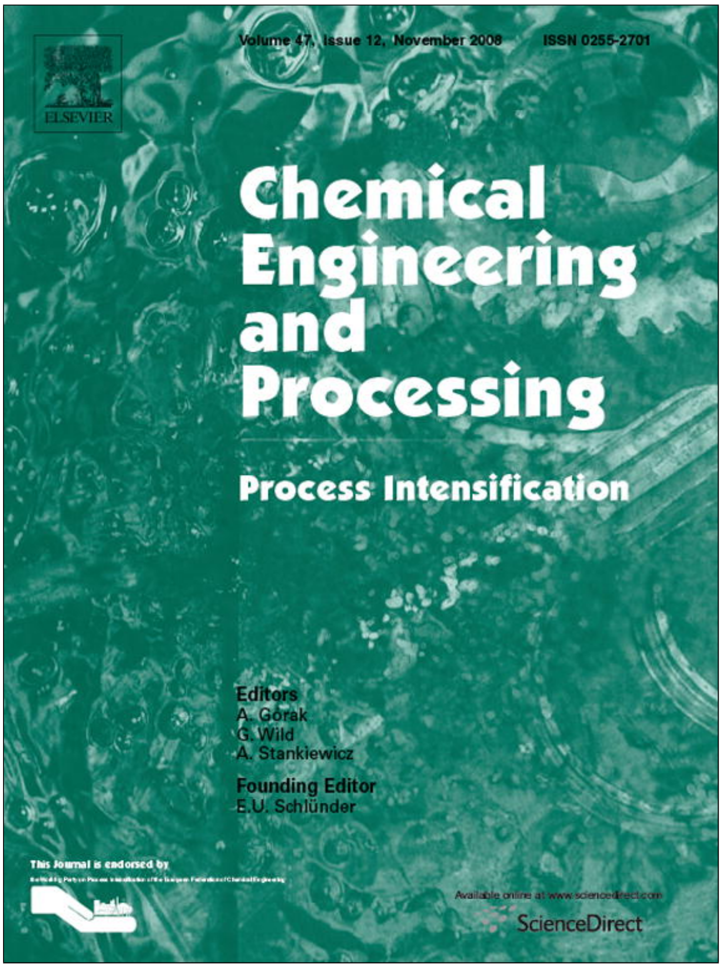
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Conditions for offset elimination in state space receding horizon controllers: A tutorial analysis

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Abstract

An offset-free control is one that drives the controlled outputs to their desired targets at steady state. In the linear model predictive control (MPC) framework, the elimination of steady-state offset may seem a little obscure, since the closed-loop optimization tends to hide the integral action. Theoretically, implementing a well-posed optimization problem and having unbiased steady-state predictions are sufficient conditions to eliminate the output offset. However, these basic conditions are not always achieved in practical applications, especially when state-space models are used to perform the output predictions. This paper presents a detailed practical analysis of the existing strategies to eliminate offset when using linear state-space models with moderated uncertainties. The effectiveness of these strategies is demonstrated by simulating three different control problems: a linear SISO system where the effect of using the estimation of the control variable is highlighted, a continuous stirred tank reactor (CSTR) with non-linear dynamics and the consequent model uncertainty and, a 2×2 system representing a distillation column that verifies the consistency of previous results and extends the conclusions to higher dimension systems.

Keywords: MPC; Predictive control; State-space models; Offset elimination; Linear control

1. Introduction

Model predictive control (MPC) technology is widely used in chemical process industries where it is generally the default technology for advanced process control applications. It refers to a class of control algorithms that optimizes future plant behavior through the use of an explicit mathematical model. Exceptional overviews of MPC and comparisons of commercial MPC controllers are available in Refs. [1–3]. The most widely used industrial MPC implementations such as DMC, QDMC, and IDCOM-M are based on stable open-loop models and cannot be used with unstable systems. To achieve offset-free control, these MPC approaches use an increment of the control variable which is computed at each sampling time (configuring an integral mode) and the predictions are corrected by adding an output step disturbance computed as the residual difference between the output measurement and the value predicted for the present time.

In this paper the analysis is restricted to those developments where the process dynamics are represented by state-space models. At each sample time, the system states are estimated and an open-loop optimization is carried out. The controller input is taken as the first element of an open-loop optimal input sequence that is computed by driving the model predicted outputs as close as possible to a desired future trajectory.

When state-space models are used, the offset elimination is accomplished in two basic ways.[1] The first approach involves working with models in their velocity form, that is, models that use input and state increments instead of input and state values. These models permit a well-posed optimization problem since the targets of the state increments are always correct (i.e., they are always zero) even if the plant and the model are not equal [4].

Abbreviations: CSTR, continuous stirred tank reactor; DMC, dynamic matrix control; MPC, model predictive control; QDMC, quadratic dynamic matrix control; IDCOM, identification command.

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[1] It is not considered here the method that modifies the controller objective including integration of the tracking error. This method augments the process model with tracking error states, and then, requires an anti-windup algorithm for the integral term [7].

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Rodrigues and Odloak [5] and Odloak [6], have reported uncommon examples of the implementation of this kind of models to MPC. In these works, the integral action is achieved by using the inputs in the incremental form in both, the output predictions and the observer. The second method involves an augmented process model including a step disturbance. This disturbance, which is estimated from the measured process variables, is generally assumed to remain constant in the future and its effect on the controlled variables is removed by shifting the steady-state target for the controller. However, when independent models are used, this second strategy has proved to be acceptable only for stable plants—it does not work for unstable systems because the observer contains the unstable poles of the process model. To overcome this problem, different authors suggested more general state-space models [7,8] considering input, state and output disturbances that allow handling unstable plants.

In this work, we focus attention on the analysis of the main existing strategies to eliminate output offset, and elucidate the critical point of some algorithms that, contrary to the appearances, cannot lead to an offset-free MPC. In addition, a comprehensive comparison between the performances of the different approaches is carried out through a few numerical simulations.

The organization of this paper is as follows. Section 2 includes a theoretical framework presentation related to three known MPC formulations: the use of the simple velocity form, the complete velocity form, and the linear regulator with disturbance sub-models. Then, Section 3 presents numeric simulations that show the improvement reached in the performance using the analyzed techniques. Finally, in Section 4, the conclusions are summarized.

2. Theoretical framework

The MPC controller uses a dynamic model of the process to predict the output trajectories and performs a constrained on-line optimization to determine the optimal future input sequence. The first control move is injected in the real plant and the procedure is repeated in the next sampling time. A significant part of the recent research literature on MPC shows contributions based on state-space models. This tendency has been stimulated by the connections found between the standard linear quadratic regulator (LQR) theory and MPC when prediction and control horizon approaches infinity and there are no constraints. In fact, an intensive effort has been done using state-space representations, for instance, on rigorous conditions for developing stable MPC [9,5,6], or to demonstrate recursive feasibility of the sequence of optimal control solutions [10].

Assuming that the MPC controller is closed-loop stable and can guide the system to a global minimum, two conditions are sufficient to obtain an offset-free control. The first one consists of getting an unbiased prediction of the steady state, which is achieved by including an integral action in the observer (provided that the control loop is based on a state-space model). This detail, in spite of its simplicity, is not always evident in the MPC formulation since the integral action is often (incorrectly) attributed to the use of velocity form models in the optimization problem. Note that, if the observer does not reach accurately the stationary states – i.e., if the observer does not include an integral action – then, the predictions are made based on a mistaken stationary model. Therefore, the optimization will lead the wrong system, not the accurate one, to the set points. As can be seen, this behavior produces a steady-state error since the inputs that eliminate the wrong-model offset are different, in general, from the ones that eliminate the offset in the true plant.

The second condition obliges to design a well-posed MPC optimization problem, and can be succinctly explained as follows [11]. Suppose that the observer gets unbiased stationary estimations, the optimization problem is not well posed (the steady-state minimum does not correspond to zero tracking error). Since the performance index is not set up so that the minimum (at steady state) corresponds to zero tracking error, then the converse must occur; that is, the optimum control will unavoidably cause offset. This is what happens when absolute input values (not the increments) are used in the cost function and the desired output is different from zero.

In the following sections, we describe standard strategies and expose the way each one accomplishes the above conditions.

2.1. Velocity form

Consider the original stable state-space model

$$x(k+1) = Ax(k) + Bu(k), \qquad y(k) = Cx(k), \tag{1}$$

where "x", "u" and "y" represent the state, the input and the model output, respectively, and A, B and C are matrices of appropriate dimensions.

A typical MPC formulation [1], that we denote Strategy 1 in this work, is based on the following cost function:

$$V_1(k) \triangleq \sum_{j=0}^{p} \left[C\,x(k+j/k) - y_{\mathrm{sp}}\right]^{\mathrm{T}} Q \left[C\,x(k+j/k) - y_{\mathrm{sp}}\right] + \sum_{j=0}^{m-1} \Delta u(k+j)^{\mathrm{T}} R\,\Delta u(k+j), \tag{2}$$

where "p" and "m" are the prediction and control horizon, respectively, "y_{sp}" stands for the output set points, $\Delta u(k) = u(k) - u(k-1)$ are the input increments, which at the same time are the optimization variables, and Q and R are positive definite weighting matrices. This cost function is minimized subject to

$$u_{\min} \le u(k+j) \le u_{\max}, \qquad \Delta u_{\min} \le \Delta u(k+j) \le \Delta u_{\max},$$

and the successive states (predictions) are computed using the current measured state $x(k)$ and the following velocity model[2]:

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix} &= \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u(k) \\ y(k) &= [\,C \quad 0\,] \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} \end{aligned}. \tag{3}$$

[2] Note that this augmented model includes an integrating mode.

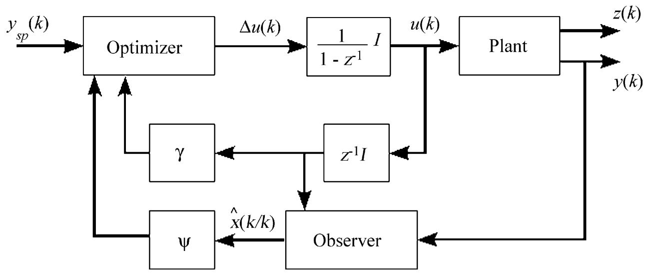

Fig. 1. Representative block diagram of the velocity form control structure. Here, $z(k)$ is a non-controlled variable and γ and Ψ are matrices defined as in Ref. [1].

The objective function (2) together with velocity models like (3), produce a well-posed optimization problem, since the combination $y=y_{\rm sp}$ and $\Delta u=0$ is always possible at steady state.

Fig. 1 shows an MPC close-loop diagram in which the block called "*optimizer*" is the one that performs the minimization described above [1]. In this diagram, matrices γ and Ψ accommodate the prediction equations taking into account the model formulation (3). Note that in (3), the input "u" represents itself a new state, which would not need to be estimated by the observer but computed by saving the implemented value.

This seems to be right, since estimating a variable that was implemented one sampling time before makes no much sense. However, as it is shown next, this is the reason why this control structure produces output offset. The trouble arises because the observer must perform explicitly the integral action. Assume that for a time $\bar{k}$ large enough, the system reaches the steady state. At this time, the output predictions will be

$$\hat{y}(\bar{k}+j/\bar{k}) = CA^j\hat{x}_{\bar{k}} + C(I+A+\cdots+A^{j-1})Bu_{\bar{k}},\quad 1\le j\le p, \tag{4}$$

where $\hat{y}(\bar{k}+j/\bar{k})$ represents the predicted output, $\hat{x}_{\bar{k}}$ represents the estimated state (that remains constant for stable observers), and $u_{\bar{k}}$ is the steady-state input. Note that, since we assume steady-state conditions, then $\Delta u_{\bar{k}}=0$. On the other hand, the observer steady-state equation is given by[3]:

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + L_{\rm aux}[y_{\bar{k}} - C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}})], \tag{5}$$

where $L_{\rm aux}$ is the observer gain, and $y_{\bar{k}}$ is the (measured) output feedback. From this equation, we can see that there is no reason for the model output, $C(A\hat{x}_{\bar{k}}+Bu_{\bar{k}})$, to achieve the plant output, $y_{\bar{k}}$. Note, from (5), that the steady-state $\hat{x}_{\bar{k}+1}=\hat{x}_{\bar{k}}$ does not mean that $\hat{x}_{\bar{k}}=A\hat{x}_{\bar{k}}+Bu_{\bar{k}}$, since the estimated states may not evolve according to matrices A and B, that are only a model. Consequently, the steady-state condition would happens with

$$\hat{x}_{\bar{k}} \ne A\hat{x}_{\bar{k}} + Bu_{\bar{k}}. \tag{6}$$

[3] Eq. (5) is derived from a typical discrete state observer.

So, the steady-state prediction equations will be given by

$$\begin{aligned}\hat{y}(\bar{k}+1/\bar{k}) &= C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}}) \ne y_{\bar{k}}\\ &\vdots\\ \hat{y}(\bar{k}+p/\bar{k}) &= CA^p\hat{x}_{\bar{k}} + C(I+A+\cdots+A^{p-1})Bu_{\bar{k}} \ne y_{\bar{k}},\end{aligned} \tag{7}$$

which means that the predicted output that the optimization will try to guide to the set point is in general different from the true stationary output. This is a case in which, despite the optimization problem is in principle well posed (that is, the minimum of the objective function corresponds to the desired effect), it is not possible to assure the output offset elimination. This is a remarkable fact since usually is supposed that working with costs that penalizes the input increments instead of the inputs, the output offset is automatically removed. In Section 3, the simulation examples illustrate this theoretical discussion.

The natural way to overcome this trouble is by adding an integrating mode to the observer. This can be made in two different forms: by including the complete model (3), that is, estimating the input "u" together with the original states; or by adding a disturbance model. Using the former of these alternatives, which we call Strategy 2, it is easy to see that the steady-state observer equation is given by

$$\begin{aligned}\begin{bmatrix}\hat{x}_{\bar{k}}\\ \hat{u}_{\bar{k}}\end{bmatrix} &= \begin{bmatrix}A & B\\ 0 & I\end{bmatrix}\begin{bmatrix}\hat{x}_{\bar{k}}\\ \hat{u}_{\bar{k}}\end{bmatrix} + \begin{bmatrix}B\\ I\end{bmatrix}\Delta u_{\bar{k}} + \begin{bmatrix}L_x\\ L_u\end{bmatrix}\\ &\quad\times\left[y_{\bar{k}} - [C\quad 0]\left(\begin{bmatrix}A & B\\ 0 & I\end{bmatrix}\begin{bmatrix}\hat{x}_{\bar{k}}\\ \hat{u}_{\bar{k}}\end{bmatrix} + \begin{bmatrix}B\\ I\end{bmatrix}\Delta u_{\bar{k}}\right)\right],\end{aligned} \tag{8}$$

where L_u and L_x form the observer gain matrix,[4] and again, $\Delta u_{\bar{k}}=0$. This equation leads to

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}} + L_x\left(y_{\bar{k}} - [C\quad 0]\begin{bmatrix}A & B\\ 0 & I\end{bmatrix}\begin{bmatrix}\hat{x}_{\bar{k}}\\ \hat{u}_{\bar{k}}\end{bmatrix}\right), \tag{9}$$

$$\hat{u}_{\bar{k}} = \hat{u}_{\bar{k}} + L_u\left(y_{\bar{k}} - [C\quad 0]\begin{bmatrix}A & B\\ 0 & I\end{bmatrix}\begin{bmatrix}\hat{x}_{\bar{k}}\\ \hat{u}_{\bar{k}}\end{bmatrix}\right). \tag{10}$$

From (10) we see that, if L_u is of full rank, then

$$y_{\bar{k}} = [C\quad 0]\begin{pmatrix}A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}}\\ \hat{u}_{\bar{k}}\end{pmatrix} = C(A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}}); \tag{11}$$

and from (9):

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}}. \tag{12}$$

Finally, from (11) and (12), predictions (7) will produce accurate outputs.

Now, provided that the closed loop is supposed to be stable and the successive optimization problem reach a minimum in steady state, it is clear that the use of "$\hat{u}(k)$" instead

[4] Observe that the observer defined in (8), in Strategy 2, is different from the one defined in (5). The input "u", is now estimated.

of "$u(k)$" in the predictions will compensate the steady-state model mismatch (or an eventual disturbance). In addition, we observe that in general is $\hat{u}_{\bar{k}} \neq u_{\bar{k}}$, which means that the additional state is only a fictitious variable with no physical meaning.[5]

It is important to remark that the use of $\hat{u}(k)$ instead of $u(k)$ is in principle counterintuitive since one could expect that the offset will be effectively eliminated when the designer use the measured input variable instead of the estimated one. In addition, to estimate a variable that is implemented by the controller (and so, it is available to be used for prediction) seems to be a meaningless decision, and that could make the designer to chose the wrong option. There are another velocity models that do not show an explicit relationship between the new state and the input u. In Ref. [5], a state is added to the original system to have an incremental model that is associated with the steady-state output. In this way, the observer automatically includes an integral action.

2.2. Complete velocity form

A different strategy to have an offset-free controller is by using the complete velocity form [12]. This kind of models considers the increments on both, the input and the states, and has the following form:

$$\varsigma(k+1) = \tilde{A}\varsigma(k) + \tilde{B}\,\Delta u(k), \quad y(k) = \tilde{C}\varsigma(k), \tag{13}$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ CB \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} 0 & I \end{bmatrix},$$

$$\varsigma(k) = \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix},$$

$$\Delta x(k) = x(k) - x(k-1).$$

On the other hand, the tracking MPC cost function can be written as

$$V_2(k) \triangleq \sum_{j=0}^{p} \left(\tilde{C}\varsigma(k+j/k) - \bar{y}_{\rm sp}\right)^{\rm T} Q \left(\tilde{C}\varsigma(k+j/k) - \bar{y}_{\rm sp}\right) + \sum_{j=0}^{m-1} \Delta u(k+j)^{\rm T} R\, \Delta u(k+j),$$

where $\bar{y}_{\rm sp} = [\,0 \quad \cdots \quad 0 \quad y_{\rm sp}\,]^{\rm T}$. Observe that in this cost the new state vector ς is directly penalized in order to achieve a stationary point in which the state increments are null and the output is equal to the set point.

Based on model (13), the output predictions will be

$$\hat{y}(k+j/k) = \tilde{C}\tilde{A}^{j}\hat{\varsigma}(k) + \tilde{C}\tilde{A}^{j-1}\tilde{B}\,\Delta u(k) + \cdots + \tilde{C}\tilde{A}^{j-m}\tilde{B}\,\Delta u(k+m-1), \quad 1 \le j \le p,$$

and taking into account that

$$\tilde{C}\tilde{A}^{j} = [\,0 \quad I\,]\begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}^{j} = [\,0 \quad I\,]\begin{bmatrix} A^{j} & 0 \\ CA^{j}+\cdots+CA & I \end{bmatrix} = [\,(CA^{j}+\cdots+CA) \quad I\,],$$

the steady-state output predictions ($k=\bar{k}$) will be

$$\hat{y}(\bar{k}+j/\bar{k}) = \tilde{C}\tilde{A}^{j}\hat{\varsigma}_{\bar{k}} = \hat{y}_{\bar{k}}, \quad 1 \le j \le p, \tag{14}$$

provided that $\Delta u = 0$ and $\Delta x = 0$ in steady state.

Eq. (14) means that the predicted output will remain constant and equal to the corresponding observer estimation. To clarify the nomenclature, note that $\hat{y}(\bar{k}+j/\bar{k})$ represents *the predictions* that will be used into the MPC optimization, while $\hat{y}_{\bar{k}}$ represents the estimate of the new state added the original model in (13). Now, we must show that the integrating mode incorporated by the complete velocity model into the observer assures that the estimation $\hat{y}_{\bar{k}}$ reaches the true plant output value. To do that, consider the observer stationary equation:

$$\begin{bmatrix} \Delta\hat{x}_{\bar{k}} \\ \hat{y}_{\bar{k}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}\begin{bmatrix} \Delta\hat{x}_{\bar{k}} \\ \hat{y}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} B \\ CB \end{bmatrix}\Delta u_{\bar{k}} + \begin{bmatrix} L_x \\ L_y \end{bmatrix}\left[y_{\bar{k}} - [\,0 \quad I\,] \times \left(\begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}\begin{bmatrix} \Delta\hat{x}_{\bar{k}} \\ \hat{y}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} B \\ CB \end{bmatrix}\Delta u_{\bar{k}}\right)\right].$$

If the estimations $\hat{x}$ and $\hat{y}$ are separated, and considering that the input and state increments are null, then

$$0 = L_x[y_{\bar{k}} - \hat{y}_{\bar{k}}], \quad \hat{y}_{\bar{k}} = \hat{y}_{\bar{k}} + L_y[y_{\bar{k}} - \hat{y}_{\bar{k}}].$$

Finally, if L_x or L_y are full rank, imply

$$y_{\bar{k}} = \hat{y}_{\bar{k}}.$$

Then, it is clear that Eq. (14) produces an unbiased steady-state output prediction. Finally, taking into account that it is assumed that the controller stabilizes the system in a stationary minimum, the output offset will be effectively eliminated. In order to organize the numeric simulation section, this methodology is designated as Strategy 3.

Before advancing to the next section, we mention an implementation detail. Both of the presented velocity models add integrating modes to the controlled system, loosing in this way the open-loop stability assumed in the original system (see (3) and (13)). Then, if it is desired to guarantee the closed-loop stability by means of an infinite prediction horizon ($p = \infty$) (this is the usual way to achieve stability in MPC, as can be seen in Ref. [9]), then the cost may become unbounded. That is, even when the input increments are null behind the control horizon "m", the

[5] Note that $u_{\bar{k}}$ is the control action implemented by the controller (so, the output offset is eliminated), while $\hat{u}_{\bar{k}}$ is the control action that uses the optimization problem to perform the output predictions.

outputs will not converge to the references[6] for time steps "k" in which the closed loop has not reached the steady state. The way to overcome this problem is by adding a set of constraints and slack variables (into the on-line optimization problem) that forces the integrating modes to be null at the end of the control horizon [5,6,10]. In this way, it is possible to show that the cost function is a Liapunov function – which guarantees the closed-loop stability – preserving the offset-free property of velocity models [10].

2.3. Linear regulator with disturbance sub-model

The use of a disturbance model as a method to include the feedback information into predictive controllers were initially proposed by Richalet et al. in Model Predictive Heuristic Control [13] and by Cutler and Ramaker in Dynamic Matrix Control [14]. In these early versions, the controllers were based on impulse and step response models, respectively, and the difference between the true plant output and the model output (that is performed by means of a simultaneous parallel model) is added to the output prediction. In this way, the output prediction reaches the true output (which is the feedback value) in the steady state.

Now, it is considered the general form in which the most recent MPC controllers eliminate the offset by means of disturbance models [7,8,15,16]. If we consider the state-space model given by (1), then, a general way to include disturbance sub-models is as follows [7]:

$$\begin{bmatrix} x(k+1) \\ d(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} A & G_d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \\ p(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(k), \tag{15}$$

$$y(k) = [\,C \quad 0 \quad G_p\,] \begin{bmatrix} x(k) \\ d(k) \\ p(k) \end{bmatrix},$$

where the state and output disturbance are $d(k) \in \mathbb{R}^{s_d}$ and $p(k) \in \mathbb{R}^{s_p}$, respectively; and G_d and G_p are matrices that determine the effect of the disturbance on the states and the output. This is a general form of considering the disturbance: if $G_d = 0$ and $G_p = I$ we have an output disturbance, on the other hand, if $G_d = B$ and $G_p = 0$ we have an input disturbance. Tacking into account the model (15), the steady-state observer equation is given by

$$\begin{aligned} \hat{x}_{\bar{k}} &= A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}} \\ &\quad + L_x[y_{\bar{k}} - C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}}) - G_p\hat{p}_{\bar{k}}] \end{aligned} \tag{16}$$

$$\begin{aligned} \begin{bmatrix} \hat{d}_{\bar{k}} \\ \hat{p}_{\bar{k}} \end{bmatrix} &= \begin{bmatrix} \hat{d}_{\bar{k}} \\ \hat{p}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} L_d \\ L_p \end{bmatrix} \\ &\quad \times [y_{\bar{k}} - C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}}) - G_p\hat{p}_{\bar{k}}], \end{aligned} \tag{17}$$

[6] Here, an optimization problem based on the open-loop predictions is solved at each sample time.

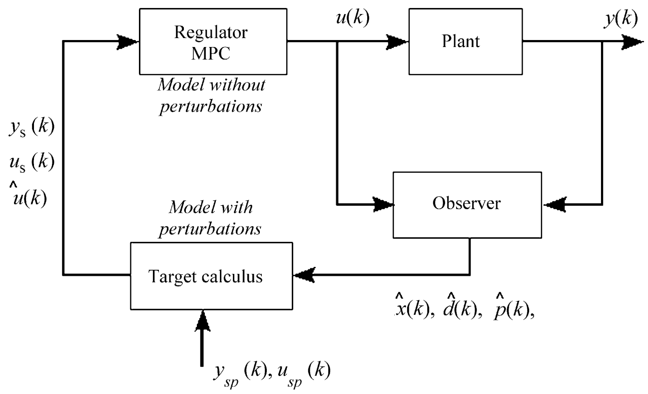

Fig. 2. Representative block diagram of the predictive control system with linear regulator (Strategy 4).

where L_x, L_d and L_p are the observer gains that corresponds to the states estimation $\hat{x}_{\bar{k}}$, and the disturbance estimation $\hat{d}_{\bar{k}}$ and $\hat{p}_{\bar{k}}$, respectively. If the system is detectable and $[\,L_d^{\mathrm{T}} \quad L_p^{\mathrm{T}}\,]^{\mathrm{T}}$ is full rank, then, from (17) we have

$$y_{\bar{k}} = C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}}) + G_p\hat{p}_{\bar{k}}.$$

Then, from (16):

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}};$$

that finally imply

$$y_{\bar{k}} = C\hat{x}_{\bar{k}} + G_p\hat{p}_{\bar{k}}. \tag{18}$$

This means that, once the observer is stabilized, the true plant output has an exact relationship with the estimated state and disturbance. This is true, naturally, because of the integral action of the observer.

Now, we analyze how the control scheme given in Fig. 2 removes the offset by means of the disturbance estimations. In this scheme (here denoted as Strategy 4), there is a MPC regulator block that drives the system to a steady state (x_s and u_s), which represent the achievable steady-state input and state target, respectively; and a Target Calculus block that is devoted to compute these steady-state values. Besides, the variables u_{sp} and y_{sp} represent the input and output set point.[7] Note that the Target Calculus stage does not take into account any economic criterion to perform the optimization. In fact, all the strategies presented in this work can be subordinated to a supervisor optimization program – as the one developed in Ref. [17] – whose main task would be the determination of economic set points for MPC.

The augmented system given by (15) has states (the states corresponding to the disturbances) that cannot be controlled by means of the input u. Then, the MPC regulator makes use of the original model (1) and shifts the steady-state targets in order to remove the estimated disturbance effect.[8] In addition, the

[7] Note this strategy allows input set points; this makes sense when the controlled system has enough degrees of freedom only.

[8] Note that to estimate the disturbances it is necessary that the augmented system be detectable. Theorems and conditions to guarantee that the disturbance-augmented systems are detectable were analyzed in Refs. [7,8]. For the

dynamic control is separated from the stationary computation: the Targets Calculus block is completely devoted to obtain the stationary values, while the MPC regulator block is dedicated to guide the states $\hat{x}(k)$ to its corresponding targets. According to [7], the optimization problem that must be solved in the regulator block is given by

$$\min_{\tilde{u}(k+j+1),\ldots,\tilde{u}(k+j+m-1)} V(k) \triangleq \sum_{j=0}^{\infty} \{\tilde{x}(k+j/k)^{\mathrm{T}} C^{\mathrm{T}} Q C \tilde{x}(k+j/k) + \tilde{u}(k+j)^{\mathrm{T}} R \tilde{u}(k+j) + \Delta\tilde{u}(k+j)^{\mathrm{T}} S\, \Delta\tilde{u}(k+j)\}$$

subject to:

$$\begin{aligned}
&\tilde{x}(k+j+1) = A\tilde{x}(k+j) + B\tilde{u}(k+j) && \forall j = 0, \infty\\
&\Delta\tilde{u}(k+j) \triangleq \tilde{u}(k+j) - \tilde{u}(k+j-1) &&\\
&\tilde{x}(k) = \hat{x}(k) - x_s &&\\
&\tilde{u}(k-1) = u(k-1) - u_s &&\\
&\tilde{u}(k+j) = 0 && \forall j \geq m\\
&u_{\min} \leq \tilde{u}(k+j) + u_s \leq u_{\max} && \forall j = 0, m-1\\
&\Delta u_{\min} \leq \Delta\tilde{u}(k+j) \leq \Delta u_{\max} && \forall j = 0, m-1
\end{aligned}$$

where it can be seen that the state and input are driven to the targets x_s and u_s, and the prediction horizon is infinite. The task of this block is to guide the shifted states and input to zero, which means that the ability of this strategy to eliminate the offset depends on the computation of the targets x_s and u_s. To perform this calculation, the following optimization problem is solved:

$$\min_{x_s, u_s} V_t(k) \triangleq \{(y_{\mathrm{sp}} - y_t^{\mathrm{a}})^{\mathrm{T}} Q_s (y_{\mathrm{sp}} - y_t^{\mathrm{a}}) + (u_s - u_{\mathrm{sp}})^{\mathrm{T}} R_s (u_s - u_{\mathrm{sp}})\} \tag{19}$$

subject to:

$$x_s = Ax_s + Bu_s + G_d\hat{d}(k) \tag{20}$$

$$y_t^{\mathrm{a}} \triangleq Cx_s + G_p\hat{p}(k) \tag{21}$$

$$u_{\min} \leq u_s \leq u_{\max},$$

where y_t^{a} is the achievable stationary output. This optimization problem is a general formulation that considers the case in which there are more outputs than inputs and some of the outputs are not controllable. If this is the case, the achievable output target y_t^{a} will be set up as close as possible (in the least square sense) to the output set point (in the case that the set point y_{sp} is achievable, then, $y_t^{\mathrm{a}} = y_{\mathrm{sp}}$). On the other hand, if the system has degrees of freedom (there are more inputs than outputs), then, the input targets will be set up as close as possible to the input set points u_{sp} in the least square sense. If the system has not degrees of freedom, then, only one solution exist to constraints (20) and (21). Note that, despite the generality of this formulation, the targets x_s and u_s must be uniquely determined in order to have a well-posed target tracking optimization problem. The conditions that must have a non-square system to achieve this uniqueness can be found in Ref. [7].

Following the strategy depicted in Fig. 2, two optimization problems must be solved at each sample time "k". Assume that constraints (20) and (21) hold true for the stationary disturbances $\hat{d}$ and $\hat{p}$ given by the observer. That is, for a time $\bar{k}$ large enough, the states and input target satisfies

$$x_s = Ax_s + Bu_s + G_d\hat{d}_{\bar{k}}, \tag{22}$$

$$y_{\mathrm{sp}} \triangleq Cx_s + G_p\hat{p}_{\bar{k}}. \tag{23}$$

On the other hand, from (17), the states and disturbances provided by the observer satisfies

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}}, \tag{24}$$

$$y_{\bar{k}} = C\hat{x}_{\bar{k}} + G_p\hat{p}_{\bar{k}}. \tag{25}$$

Then, subtracting (22) from (24) we have[9]

$$(\hat{x}_{\bar{k}} - x_s) = A(\hat{x}_{\bar{k}} - x_s) + B(u_{\bar{k}} - u_s), \tag{26}$$

which corresponds to the original system considered by the target tracking optimization. If the original model given by A, B and C can be stabilized, then the regulator will guide the states $\tilde{x}(k)$ to zero; that is[10]

$$(\hat{x}_{\bar{k}} - x_s) = 0. \tag{27}$$

Now, subtracting (23) from (25), we have

$$(y_{\bar{k}} - y_{\mathrm{sp}}) = C(\hat{x}_{\bar{k}} - x_s), \tag{28}$$

which finally, from (27) and (28), implies

$$y_{\bar{k}} = y_{\mathrm{sp}}.$$

Note, finally, that the use of an augmented model requires the simultaneous solution of both optimization problems. This can increment the computational cost of the algorithm, especially if the system has a large number of input and outputs.[11]

augmented system (15), these conditions are: the original system given by (A, C) is detectable, and rank $\begin{bmatrix} (I-A) & -G_d & 0 \\ C & 0 & G_p \end{bmatrix} = n + s_d + s_p$, where n is the original states dimension and s_d and s_p are the dimension of the disturbance states.

[9] Note that in Fig. 2 the observer passes the estimated disturbances to the Target Calculus block, and then, the disturbances are identical in both blocks.

[10] Note that once the closed loop reach a stationary state ($k = \bar{k}$), the dynamic predictions in the cost function become: $\tilde{x}(\bar{k}) = \hat{x}_{\bar{k}} - x_s$; $\tilde{x}(\bar{k}+1/\bar{k}) = A\tilde{x}(\bar{k}) + B\tilde{u}(k) = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} - (Ax_s + Bu_s) = \underbrace{\hat{x}_{\bar{k}} - G_d\hat{d}_{\bar{k}}}_{\text{from (24)}} - \underbrace{(x_s - G_d\hat{d}_{\bar{k}})}_{\text{from (22)}} = \hat{x}_{\bar{k}} - x_s; \ldots \tilde{x}(\bar{k}+j/\bar{k}) = \hat{x}_{\bar{k}} - x_s$. Then, $C\tilde{x}(\bar{k}+j/\bar{k}) = C(\hat{x}_{\bar{k}} - x_s) = C\hat{x}_{\bar{k}} - Cx_s = \underbrace{y_{\bar{k}} - G_p\hat{p}_{\bar{k}}}_{\text{from (25)}} - \underbrace{(y_{\mathrm{sp}} - G_p\hat{p}_{\bar{k}})}_{\text{from (23)}} = y_{\bar{k}} - y_{\mathrm{sp}}$.

[11] This fact contrasts with the simpler case of using velocity models, in which only one optimization problem is solved at each sample time.

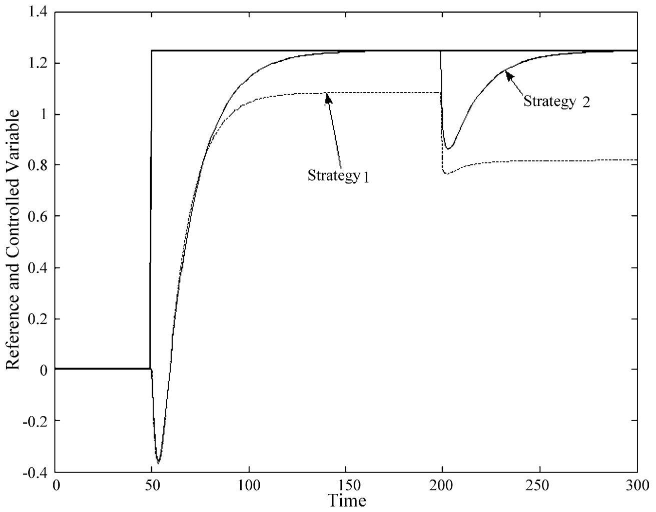

Fig. 3. Step responses to set point and load changes when the Strategies 1 and 2 are implemented.

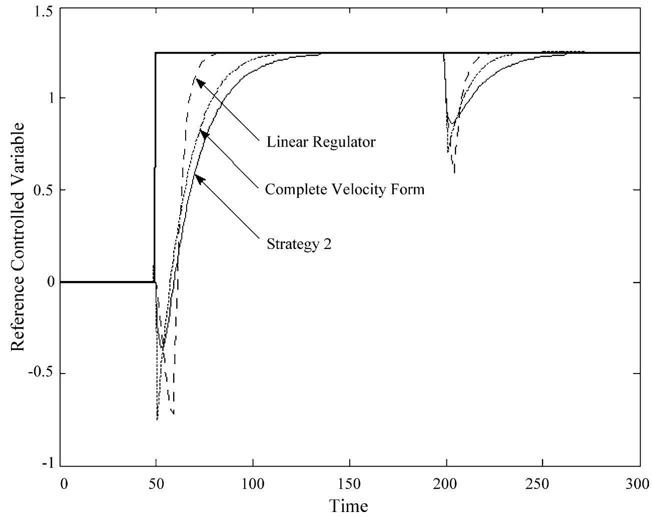

Fig. 4. Step responses to set point and load changes when the Strategies 2, 3 and 4 are implemented.

3. Numeric simulations

In this section, three application problems are simulated that show the behavior attainable with the described formulations. Note that this verification must be done in the presence of model uncertainties or load disturbances; otherwise the offset does not appear. First, a SISO linear case with gain uncertainty is simulated to observe the role the observer plays in the offset elimination. Then, a non-linear case – a CSTR with significant parametric uncertainty – is tested, where the results confirm the expected offset elimination property when the controller has the appropriate structure. Finally, a MIMO (2×2) system representing a distillation column is used to verify the consistency of previous results and confirm that previous conclusions can be extended to higher dimension systems.

3.1. Linear case

Consider a linear plant given by

$$G(s) = \frac{K(-9s+1)}{45s^2 + 18s + 1}, \tag{29}$$

where $K=1$ for the true plant, and $K^0=0.85$ for the nominal plant. In addition, an output load variable is included in order to consider load disturbances besides set point changes.

Fig. 3 shows the step response (time interval from 50 to 200) when the Strategy 1 and the corresponding reformulation (Strategy 2) are implemented. Note that in the velocity form given in Section 2.1 the offset is not null after the transitory response has ended, while the second strategy has completely eliminated the error. The responses to the load disturbance show that the offset of Strategy 1 increases as compared with the previous one.

On the other hand, Fig. 4 shows the time response given by Strategies 2, 3 and 4 to changes in set point and load. In the three cases, the offset is completely eliminated. The inspection of the results in Fig. 4 tells us that all the discussed strategies produce quite reasonable offset-free responses. In particular, the linear regulator seems to be more appropriate than the other ones when the settling time is taken as a critical feature of the desired response. Note that a fair comparison between the analyzed treatments should involve a study of the individual tuning procedures, which is out of the scope of this paper. All the controllers studied in this work have been implemented using $Q=500I_{p\times 1}$, $R=0.5I_{m\times 1}$, $p=25$ (prediction horizon), $m=3$ (control horizon), $u_{\max}=15$ and $u_{\min}=-8$. The adopted sampling interval is $T_s=15$ s, and the observers have been designed with poles at $z=0.1$.

Now, it is interesting to show the behavior of the actual input $u(k)$ implemented by the controller and the estimated input $\hat{u}(k)$ provided by the augmented observer in (8), when Strategy 1 and 2 are used. Fig. 5 shows important stationary differences between both responses, clarifying the cause of the offset when a single observer (like the one presented in (5)) is applied.

In addition, the difference between the estimated state $\hat{x}(k)$ and the predicted state $A\hat{x}(k) + Bu(k)$ can be seen when both, the

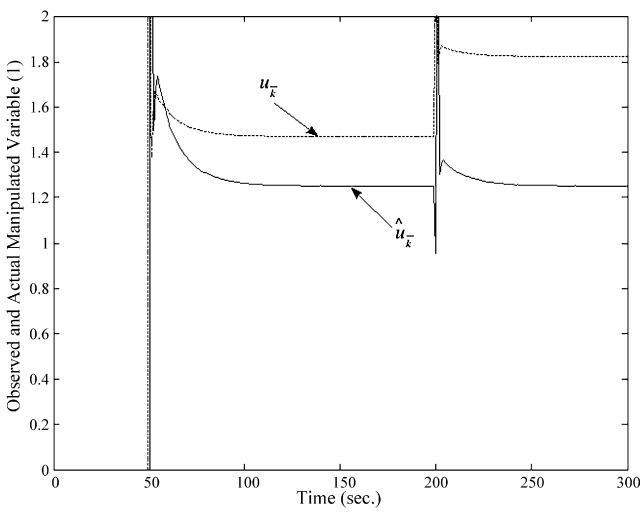

Fig. 5. Implemented (actual) input $u(k)$ and estimated input $\hat{u}(k)$.

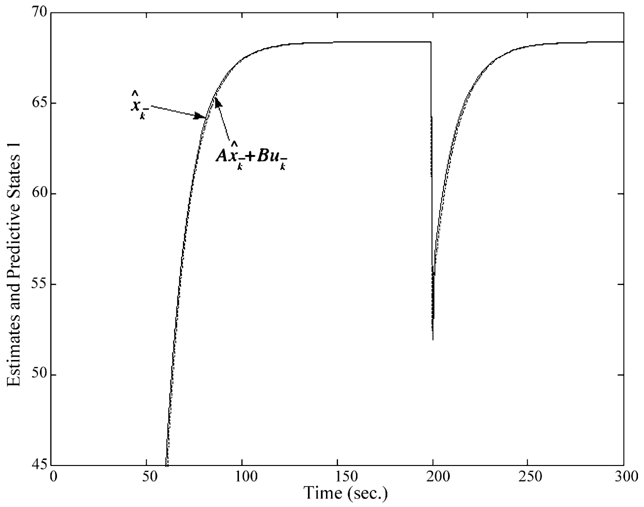

Fig. 6. Estimated state $\hat{x}(k)$ and predicted state $A\hat{x}(k) + Bu(k)$ (using the actual input $u(k)$).

implemented input $u(k)$ and the estimated input $\hat{u}(k)$, is used to make the computation. This is depicted in Figs. 6 and 7, where only the first predicted state is shown. Note that, in the first case, provided that the observer equation has not an explicit integral action, the predicted steady state cannot be stabilized in the estimation $\hat{x}(\bar{k})$; in other words, $A\hat{x}(\bar{k}) + Bu(\bar{k}) \neq \hat{x}(\bar{k})$, which means that the steady-state output predictions will show an important mismatch with respect to the actual output value.

3.2. Non-linear case

A CSTR with a strong non-linear dynamic [18] is considered now. The goal is to control the reactor temperature in presence of set-point changes and unexpected disturbances (feed flow rate and temperature). Fig. 8 shows the output responses of the Strategies 1 and 2 when a step change in set point is introduced. Clearly, Strategy 1 gives offset while the Strategy 2 reaches the new reference signal.

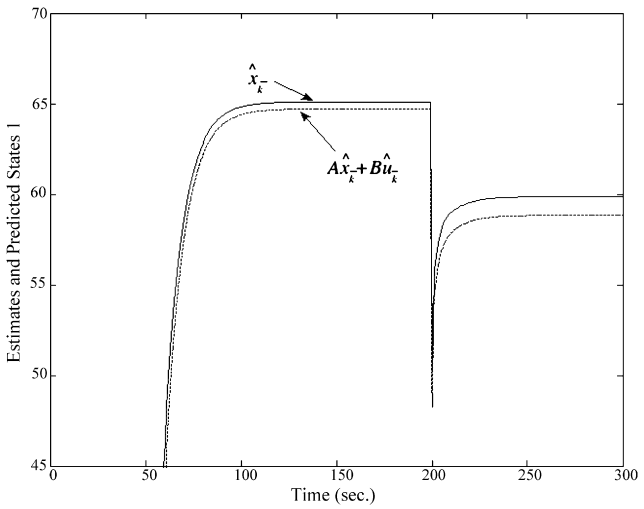

Fig. 7. Estimated state $\hat{x}(k)$ and predicted states $A\hat{x}(k) + B\hat{u}(k)$ (using the estimated input $\hat{u}(k)$).

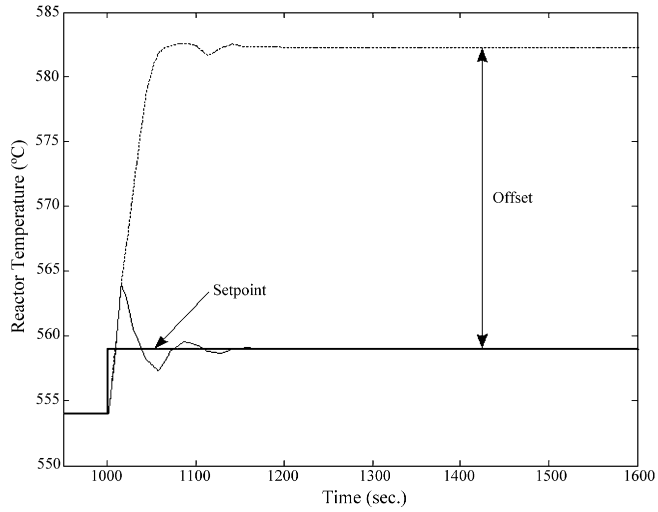

Fig. 8. Output responses to set point changes when the Strategies 1 and 2 are implemented.

Fig. 9 shows the time responses of Strategies 1 and 2 when load step changes (10% in feed flow and 10% in feed temperature) are introduced. Again, the Strategy 1 gives offset whereas the Strategy 2 completely rejects the load changes.

Fig. 10 shows the time responses reached for Strategies 2, 3 and 4. Clearly, the three strategies eliminate the output offset when a set point change is introduced. The same result is verified when a load change (10% in feed flow) is introduced into the CSTR (Fig. 11). In this example it is not possible to use both kind of disturbances $d(k)$ and $p(k)$ simultaneously when Strategy 4 is implemented, since there is only one output. For this application example, the transfer function matrices G_p and G_d were identified using a regular least squares technique, this shows to be appropriate to achieve the main objective of eliminating output offset. Note that, since the MPC regulator does not use the augmented model (it is only used by the observer and the target calculation), the output performance is not strongly dependent of matrices G_p and G_d.

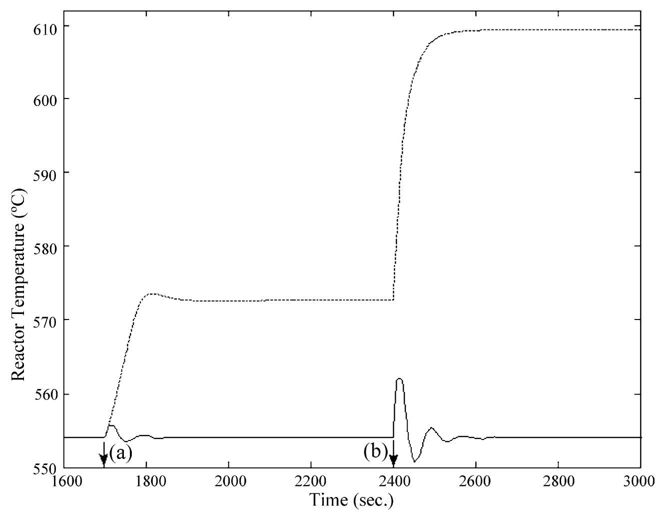

Fig. 9. Output responses using Strategies 1 and 2 to (a) a 10% step change in feed flow and (b) a 10% step change in feed temperature.

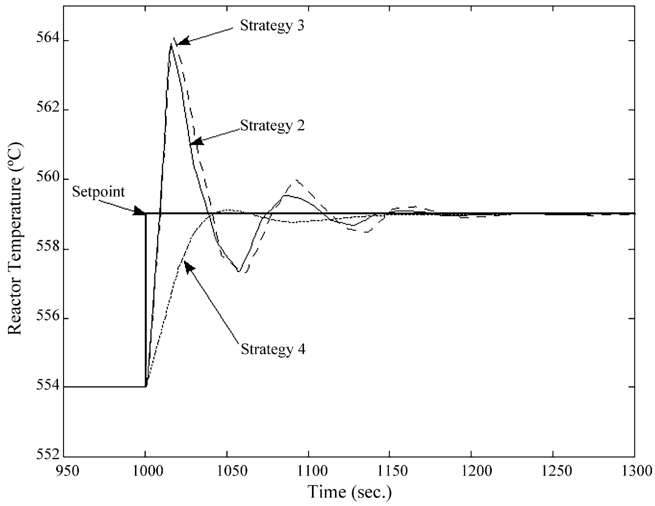

Fig. 10. Output responses to set point changes using Strategies 2, 3 and 4.

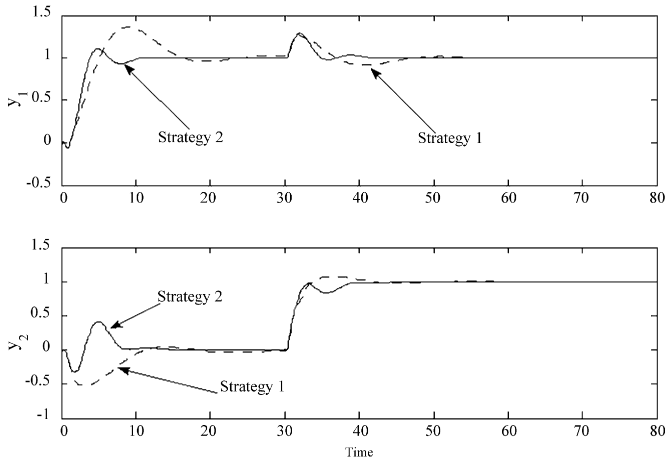

Fig. 12. Step responses to set point changes when Strategies 1 and 2 are implemented.

3.3. MIMO case

Let us analyze now a two-by-two system representing a distillation column [19], whose transfer function matrix is given by

$$G(s) = \begin{bmatrix} \dfrac{-2.2\,e^{-s}}{7s+1} & \dfrac{1.3\,e^{-0.3s}}{7s+1} \\ \dfrac{-2.8\,e^{-1.8s}}{9.5s+1} & \dfrac{4.3\,e^{-0.3s}}{9.2s+1} \end{bmatrix}.$$

This system is known in the literature as the Vinante–Luyben (VL) column. All the strategies analyzed previously were applied to this system to detect problems to achieve offset-free steady states in multivariable systems. Part of the test was done by applying unit step changes in set-points for y_1 (at $t=0$) and y_2 ($t=30$). And, since in this case no difference was specified between plant and model, load changes in the manipulated variables were produced at $t=0$ for u_1 and at $t=40$ for u_2, in order to observe the effect of the uncertainty implicit in these disturbances.

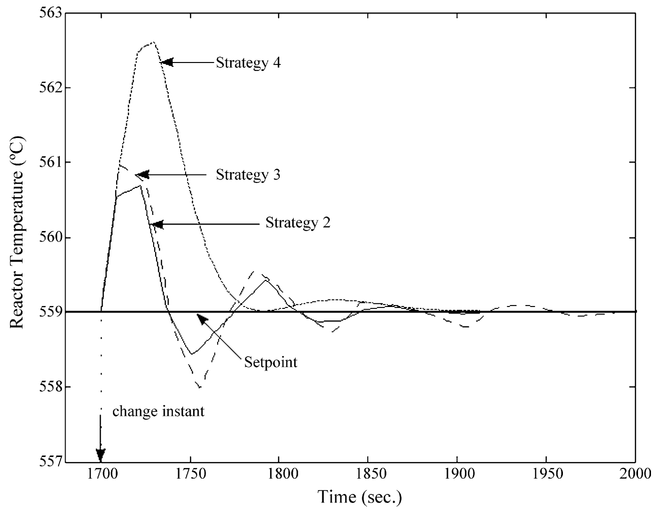

Fig. 11. Output responses of the Strategies 2, 3 and 4 to a 10% step change in feed flow.

Fig. 12 shows the set point responses when Strategies 1 and 2 are implemented. The simulations show that using Strategy 2 it is possible to obtain smaller overvalues and undervalues than with Strategy 1. Note, as a difference with the first application example presented here, that none of the strategies gives output offset; this is because no uncertainties are considered in the model. On the contrary, Fig. 13 shows that an important offset is produced when Strategy 1 is implemented; despite model uncertainties are not considered, the offset appears because the disturbance is introduced in an input variable. This disturbance has the same effect of an additive uncertainty in the transfer function matrix, which is not detected by the observer since it uses the controller output instead of the complete information of the input variable.

Figs. 14 and 15 show responses of the analyzed strategies to set-point changes and input disturbances, respectively, verifying that Strategies 3 and 4 eliminate the offset completely. Notice that for this particular case of the VL column, a better performance is obtained with the Strategy 4 than with the Strategy 3 due to the responses is less oscillatory.

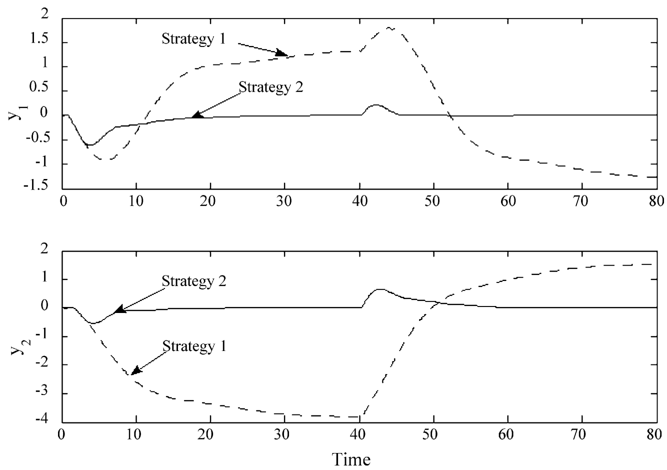

Fig. 13. Disturbance responses to step changes in the manipulated variables when Strategies 1 and 2 are implemented.

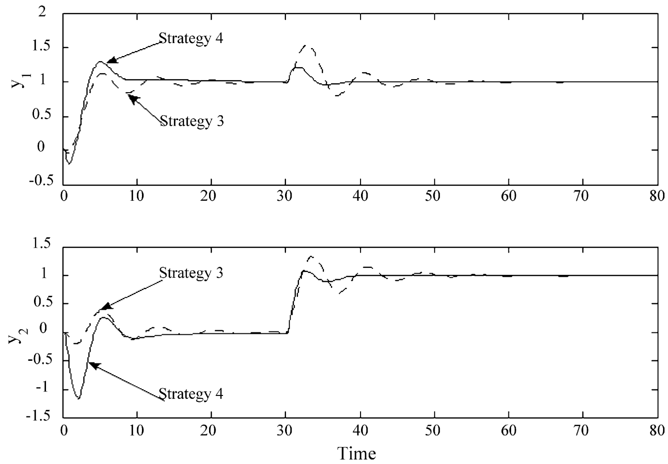

Fig. 14. Step responses to set point changes when Strategies 3 and 4 are implemented.

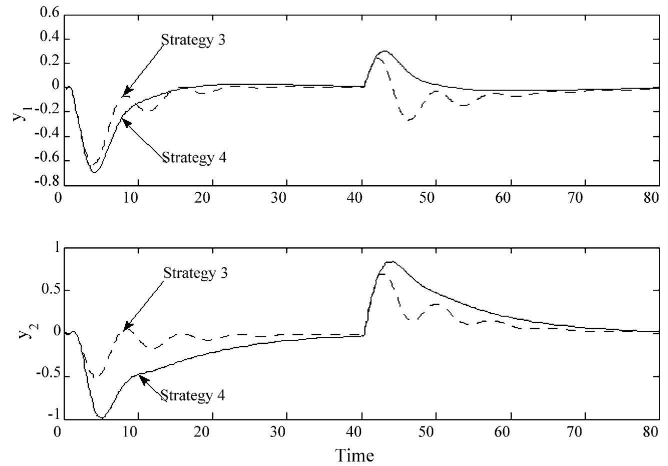

Fig. 15. Disturbance responses to step changes in the manipulated variables when Strategies 3 and 4 are implemented.

4. Conclusions

The usual way to guarantee the offset elimination in MPC is by means of an augmented model that includes disturbances. When state-space models are used, a target calculation is needed to compute the steady-state targets that, by means of the disturbance estimates, remove their influences from the controlled variables. An alternative way to achieve an offset-free MPC controller is by means of the use of velocity models. Despite in this case the state dimension is increased, there is no need to compute steady-state targets, which represents an important advantage. One of the reasons why this method is not often employed is that, in principle, a problem would arise when an infinite prediction horizon is used to guarantee closed-loop stability. This happens because the integrating states that contain the velocity models may produce a permanent output error, which may cause an unbounded cost. However, it was shown by Odloak [6], that this problem could be solved using terminal constraints and appropriate slack variables.

Based on the results obtained from running different simulation examples, it is possible to conclude the following:

(i) An adequate reformulation of the classical velocity-form model (Section 2.1) can lead to a free offset time response.
(ii) Offset-free MPC controllers are not always simple to design, particularly when velocity-form models (Strategy 2) are used to predict the output behavior. In this case, the augmented state must be estimated despite it represents a variable that can be measured.
(iii) The resulting controller yields offset-free responses only in case that the prediction is unbiased (which implies the use of an observer with integrating mode) and the optimization problem is well posed.

These are important features to be considered when designing a MPC controller.

Acknowledgements

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Appendix A. Nomenclature

List of symbols

A,B,C	state-space model matrices
d	state disturbance
G	disturbance gain matrix
$G(s)$	transfer function
I	identity matrix
j	index of prediction number
k	discrete time variable
K	transfer function gain
L	observer gain
m	control horizon
n	dimension of the state vector
p	prediction horizon
Q,R,S	positive definite weighting matrices
s	Laplace domain operator
s_d	dimension of the state disturbance
s_p	dimension of the output disturbance
T_s	sampling interval (s)
u	control variable
u_{max}	maximum control value
u_{min}	minimum control value
V	quadratic cost function
x	state variable vector
y	output variable vector
z	discrete domain operator

Greek letters

γ	matrix of the predictor [1]
Δ	increment operator
ζ	state variable of the augmented model
Ψ	matrix of the predictor [1]

Subscripts

d	state disturbance

k discrete time indicator
p output disturbance
s steady-state target
sp set point
t computed at time t
u input
x state
y output

Superscripts
$\hat{}$ prediction
$-$ steady-state condition
$\sim$ augmented model
a achievable value
T transpose matrix

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