FEEDFORWARD CONTROLLER DESIGN BASED ON H_∞ ANALYSIS

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Abstract. This paper presents a model based design and adjustment procedure for feedforward controllers that accounts for model uncertainties. It also analyses relevant properties of the feedforward-feedback structure and proposes a combined design method using robust control concepts in the frequency domain. This approach allows the inspection of the performance limits achievable for each particular application. The benefits of the suggested approach are discussed and illustrated through an application example.

Keywords: feedforward controller, model-based control, H_∞ design.

1. Introduction

There are many examples from different engineering areas where an input disturbance has a strong effect on the controlled variable. Classic methods for tuning feedback controllers like Ziegler and Nichols (1942) among others have been used per decades. However, the level of performance obtained in set-point changes is not always reached in disturbance rejections. Moreover, the combined feedforward-feedback control scheme considers simultaneously both the set-point tracking and the regulation problems. The use of this combined control is usually very common in the process industry: it can be found in distillation columns (Rix *et al.*, 1997), power plants (Weng and Ray, 1997; Mcaburn and Hughes, 1997), continuous reactors, among other examples. In spite of this, the synthesis of feedforward controllers has not received much attention. Consequently, the feedforward controller design and adjustment still follows classic approaches (Seborg *et al.*, 1989; Stephanopoulos, 1984).

In this work, a method based on H_∞ design concepts is proposed which complements the classic design by providing a rational procedure to achieving a realizable controller transfer function. In order to present systematically this work, Section 2 gives the preliminary concepts related to feedforward-feedback control, Section 3 presents main theoretical fundaments and includes an application example that reveals the capabilities of the proposed design technique. Finally, in Section 4, the conclusions are presented.

2. Feedforward-Feedback Control

Let us start by analyzing the classical feedforward-feedback structure where linear representations with uncertain parameters are assumed for the process system.

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Figure 1 shows an sketch of the combined control system where $c(s)$ is the feedback controller, $c_f(s)$ is the feedforward controller, $p_d(s)$ stands for the transfer function between the output $y(s)$ and the exogenous disturbance $d(s)$ and, $p(s)$ is the transfer function between the output and the manipulated variable $u(s)$. From this representation it is clear that $p_d(s)$ and $p(s)$ are lineal time invariants (LTI) models representing the real plant.

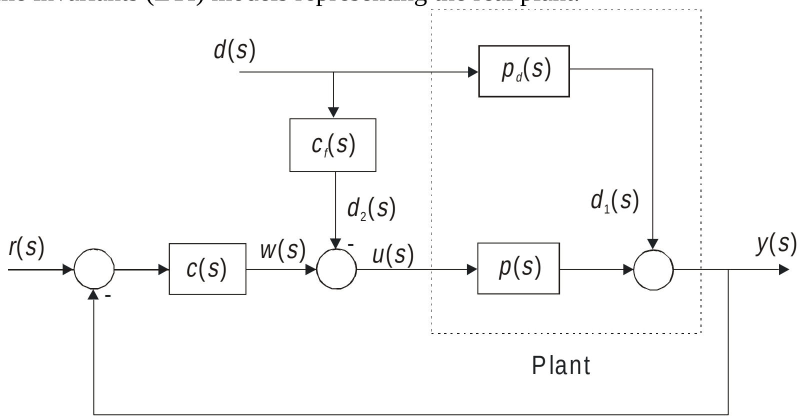

Fig. 1. Schematic representation of a feedforward-feedback control scheme.

When the feedforward controller is implemented alone, the disturbance effect on the output variable is written as

$$y(s) = \left[p_d(s) - c_f(s) p(s) \right] \; d(s) \quad \textbf{(1)}$$

but in case that a feedback loop is also used,

$$y(s) = \frac{p_d(s) - c_f(s) p(s)}{1 + p(s) c(s)} d(s) \quad \textbf{(2)}$$

In both cases, the optimal feedforward controller that minimizes $\left\| p_d - c_f p \right\|_2$ is,

$$c_f(s) = \frac{p_d(s)}{p(s)} \quad \textbf{(3)}$$

and it would lead to prefect disturbance compensation in both cases if the $c_f(s)$ is realizable. However, the expression **(3)** might result in an improper or unstable transfer function; when this happens $c_f(s)$ should be chosen such to minimize the effect of the disturbance on the controlled variable.

2.1. Properties of the Feedforward-Feedback Control System

Let us assume inputs $d(t)$ and $r(t)$ are bounded signals, i.e., $d(t)$, $r(t) \in L_2[0,\infty)$, where $L_2[0,\infty)$ stands for any continuous signals on $[0,\infty)$ that have finite 2-norm (Green and Limebeer, 1995).

Definition 1 *(Perfect Disturbance Rejection). Feedforward control allows perfect disturbance rejection when the controlled variable does not change as consequence of any bounded input disturbance, i.e.,*

$$y(t) = 0 \quad \forall \; t \geq 0 \;\; for \;\; d(t) \in L_2[0,\infty) \;\; \text{and} \; r(t) = 0 \quad . \quad \textbf{(4)}$$

Equation **(4)** assumes the output variable $y(t)$ is defined as a deviation from the steady-state value. Note that, feedforward control allows the possibility of perfect disturbance rejection while; this is not possible using feedback control neither for regulation nor for tracking problems (Morari and Zafirou, 1989).

In order to analyze the internal stability of the combined feedforward-feedback control system the following definition is introduced:

Definition 2 *(Internal Stability of the Combined Control System). The system sketched in Fig. 2 is internally stable if the elements of the transfer matrix*

$$M = \begin{bmatrix} S(p_d - c_f\, p) & S\,c\,p \\ -S\,c(p_d - c_f\, p) & S\,c \\ p_d & 0 \\ c_f & 0 \end{bmatrix} \quad with \;\; S := 1/(1 + c\,p) \qquad \textbf{(5)}$$

from $[d \;\; r]^T$ *to* $[y \;\; w \;\; d_1 \;\; d_2]^T$ *belong to* $\Re H_\infty$, *for all bounded* $d(t)$ *and* $r(t)$.

Note that requiring internal stability to the feedback loop is not sufficient for the combined system. To make sure internal stability it is also necessary that $c_f(s)$ and $p_d(s)$ being stable plants. Furthermore, if this condition is satisfied the internal stability of the feedback loop is not affected since; (i) all the input signals to the feedback loop are bounded and (ii) the feedforward controller is not included in the characteristic equation.

In order to generalize the analysis, let us assume that this plant can be described by two families of models which are defined using the classic multiplicative uncertainty,

$$\Pi_u := \left\{ p : \left| lm_u(j\omega) \right| := \left| \frac{p(j\omega) - p_0(j\omega)}{p_0(j\omega)} \right| \le \overline{lm_u}(\omega) \right\} \quad , \qquad \textbf{(6)}$$

$$\Pi_d := \left\{ p_d : \left| lm_d(j\omega) \right| := \left| \frac{p_d(j\omega) - p_{d0}(j\omega)}{p_{d0}(j\omega)} \right| \le \overline{lm_d}(\omega) \right\} \quad . \qquad \textbf{(7)}$$

Π_u and Π_d stand for the families, $\overline{lm_u}(\omega)$ and $\overline{lm_d}(\omega)$ are the upper bounds for the multiplicative uncertainty modulus $\left| lm_u(j\omega) \right|$ and $\left| lm_d(j\omega) \right|$ respectively. Thus, any member of the families of plants Π_u and Π_d can be represented as $p(s) = p_0(s)(1 + lm_u(s))$ and $p_d(s) = p_{d0}(s)(1 + lm_d(s))$ respectively and, $p_0(s)$ and $p_{d0}(s)$ are the nominal plants according to the designations in Fig. 1.

Then, using the sensitivity function $S = 1/(1 + cp)$ and substituting $p(s)$ and $p_d(s)$ in the expression **(2)** gives,

$$\frac{y(s)}{d(s)} = S(s) \; \left\{ p_{d0}(s)\left[1 + lm_d(s)\right] - c_f(s)\, p_0(s)\left[1 + lm_u(s)\right] \right\} \quad . \qquad \textbf{(8)}$$

Based on the right side of this expression, a functional denoted I_{ff} is defined as follows:

$$I_{ff}(s) := p_{d0}(s)\left[1 + lm_d(s)\right] - c_f(s)\, p_0(s)\left[1 + lm_u(s)\right] \quad . \qquad \textbf{(9)}$$

Note that if, $\left| S(j\omega) I_{ff}(j\omega) \right| \le \gamma$ for all $0 \le \omega < \infty$ then, according to previous definition and Eqn. **(2)**, it ensures that, $\left\| y(j\omega) \right\| \le \gamma \left\| d(j\omega) \right\|$.

It is important to remark that, i) $S(s)$ does not depend of feedforward controller parameters and, ii) an acceptable disturbance attenuation in the output variable is reached if $\gamma << 1$. That is, the appropriate design of both feedforward and feedback controllers should be such that a γ << 1 be reached. Furthermore, according to the comments following to definition 2, $c(s)$ should assure stability of the feedback loop and $c_f(s)$ should be stable.

If the nominal plant satisfies the performance objectives, the system reaches the nominal performance. This idea can be expressed for the feedforward-feedback control loop based on the expression **(2)** as, $\left\| W_p S_0 I_{ff\,0} \right\|_\infty \leq 1$, where $\| \, \|_\infty$ denotes the H infinity norm based on the classical definition, S_0 is the nominal sensitivity function defined as $S_0 := 1/(1+cp_0)$, $I_{ff\,0} := p_{d\,0} - c_f\, p_0$ and W_p is a weight of the input signal included for a more general formulation.

On the other hand, $\left| S(j\omega) I_{ff}(j\omega) \right| \leq \gamma$ is a robust performance condition for the feedforward-feedback control system if the parameter γ is chosen less than one and the closed-loop system is robustly stable. This condition can be formalized as follows, $\left\| W_p S\, I_{ff} \right\|_\infty \leq 1$.

Lemma 1. *Let $p(s)$ and $p_d(s)$ be plants that belong to the families* **(6)** *and* **(7)** *respectively, then, the feedforward-feedback control system reaches robust performance if the following condition is satisfied:*

$$\left| W_p(j\omega) S_0(j\omega)\, I_{ff}(j\omega) \right| + \left| T_0(j\omega) lm(j\omega) \right| \leq 1 \quad \forall\, 0 \leq \omega < \infty \quad . \tag{10}$$

The proof, given in Adam and Marchetti (2001), starts from the condition $\left\| W_p S\, I_{ff} \right\|_\infty \leq 1$ and consequently each element of the set **(6)** and **(7)** satisfies Eqn. **(10)**.

3. Feedforward Controller Design

3.1. The Standard Design Problem

The feedforward controller can be synthesized according to the following equation:

$$c_{f\,0}(s) = \left\{ p_{d\,0}(s) / p_0(s) \right\}_* \quad , \tag{11}$$

where $p_{d0}(s)$ and $p_0(s)$ are nominal transfer functions and $\{\}_*$ denotes a polynomial ratio such that it does not include: i) the zeros of $p_0(s)$ that belong to left hand side plane, ii) the unstable poles of $p_{d0}(s)$ and iii) positive time delay that might result from the difference $\theta_{d0}(s)$ - $\theta_0(s)$ where, $\theta_{d0}(s)$ and $\theta_0(s)$ are the nominal plant delays of $p_{d0}(s)$ and $p_0(s)$ respectively.

The operation indicated in **(11)** gives a stable transfer function, in spite of $p_{d0}(s)$ could be unstable and $p_0(s)$ could have right half zeros, but in many cases a non realizable transfer function could result. Hence, a filter is included in order to obtain realizability. This gives

$$c_f(s) = c_{f\,0}(s)\; f(s) \quad , \tag{12}$$

where the filter is defined as follows:

Definition 3 *(Feedforward Filter). The feedforward filter $f(s)$ is chosen as a rational transfer function,*

$$f(s) := \frac{K_f}{(\lambda\, s+1)^n} \quad , \tag{13}$$

with $n = -\delta(\mathrm{RO})\left[p_{d\,0}(s) / p_0(s) \right]$.

The function δ is defined as, $\delta = \delta(RO) := 1$ if $\mathrm{RO}\left[p_{d\,0}(s)/p_0(s) \right] < 0$ or $\delta = \delta(RO) := 0$ if $\mathrm{RO}\left[p_{d\,0}(s)/p_0(s) \right] \geq 0$ and, RO[•] is an operator that computes the relative order of the polynomial ratio contained inside the brackets. In other words, it gives the difference between the denominator polynomial order and the numerator polynomial order.

3.2. Proposed Design Technique

Since, $c_f(s)$ has the gain and the filter time constant as adjusting parameters, the following optimization problem in the frequency domain is proposed:

$$\min_{K_f,\lambda} \left\| S(j\omega) I_{ff}(j\omega) \right\|_\infty \quad \forall \;\; 0 \le \omega < \infty \tag{14}$$

s.t., $K_f > 0$ and $\lambda \ge 0$.

3.3. Example

Consider the boiler with natural recirculation modeled by Adam and Marchetti (1999). Adam (1996) demonstrates that for the operative conditions the non-linear dynamic model between level drum and the feed-water can be satisfactorily modeled by the linear expression $p_0(s) = 1.9510^{-4} e^{-6.1s} / s(5.72s+1)$. Similarly, the transfer function between the level drum and the steam load is $p_{d0}(s) = 1.5810^{-4}(75.49s-1)/s(6.88s+1)$. According to the feedforward controller design proposed in this paper

$$c_f(s) = 0.810 K_f (75.49s - 1)/(\lambda s + 1) \quad . \tag{15}$$

Furthermore, using the extreme plant concept, it is possible to define the multiplicative uncertainties $lm_u(s)$ and $lm_d(s)$ according to Eqn. (6) and (7). Thus, any plant $p(s) \in \Pi_u$ or $p_d(s) \in \Pi_d$ is included in the Nyquist plane inside the extreme plants $\bar{p}(s) = 2.1810^{-4} e^{-12.17s} / s(1.55s+1)$ or $\bar{p}_d(s) = p_{d0}(s) e^{-5s}$ respectively.

Figure 2 shows the $\left| S(j\omega) I_{ff}(j\omega) \right|$ when a PI feedback controller tuned by the TL method (Tyreus and Luyben, 1992) is used (Adam, 1996). Applying the optimization problem proposed in Section 3, the optimal parameters for the feedforward filter are K_{fot} = 2.22 and λ = 3.11 and, $\left\| SI_{ff} \right\|_\infty = 1.05710^{-5}$ at frequency ω = 0.6136.

Figure 3 compares the level control performance reached when the feedback and the combined scheme are used. For the last case, two feedforward controller are studied, the first one is tuned by the proposed method and the other one is tuned as a static feedforward controller calculated for s = 0, according to classic textbook. In every case, a PI feedback controller tuned by the TL method was adopted. Clearly, it is observed the improvement reached on the controlled variable when the proposed feedforward controller is used in comparison with the other cases.

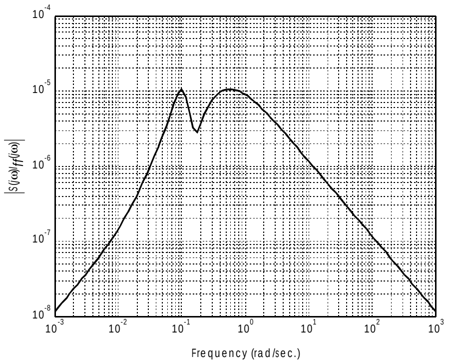

Fig. 2. $\left|S(j\omega)I_{ff}(j\omega)\right|$ using the PI controller tuned by TL method.

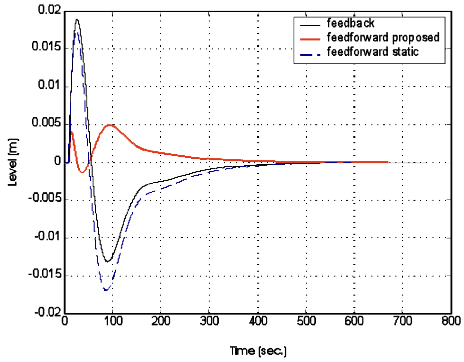

Fig. 3. Level control using the feedback and feedforward-feedback with static and proposed feedforward controllers.

4. Conclusions

In this work, a study of the feedforward-feedback control configuration is presented. Under the framework of the robust control theory, a method is proposed to achieve the realizable feedforward controller by including of a low pass filter. The adjustment of this filter is done by a minimization of an objective functional in the frequency domain and based on H_∞ design concepts that take into account the model uncertainties.

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